”A Testbed for Networked Control Systems”

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Project Report

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July 27, 2011

Table of Content

1 Introduction

This chapter introduces the background, purpose and results of the project.

1.1 Background

The primary purpose of the project was to develop a test bed that can facilitate research on time scales theory and networked control theory.

Time Scales theory is a way to unify the seemingly disparate fields of discrete dynamical systems (i.e., difference equations) and continuous dynamical systems (i.e., differential equations). Time scale systems might best be understood as a “bridge” between discrete time and continuous time systems. A networked control system is a control system that uses a shared network to transmit information. It is widely used in today’s life.

To read this project report, it is assumed that the reader has a basic knowledge of control theory, time scales theory, and networked control systems. Please refer to Chapter 3 of [1], Chapter 1 of [2], and [3] for detailed information.

1.2 What Has Been Done

In this project we developed a test bed for modeling and simulating the time scales control and networked control system for a rotary inverted pendulum [4]. The test bed, which allows users to develop, simulate and run their own control algorithms, could be used to implement and verify various theories.

The test bed provides both simulation and executable models for a rotary inverted pendulum. The simulation models, for both the local control system and the network control system, were built using Matlab/Simulink [5]. The simulation environment enables a user to verify the control algorithms. The Quanser QUARC library [6] was used to create an executable model which could directly access the hardware components. This executable model can be compiled and run on QNX [7]. To implement a time scales control, a user could choose to use an external signal generator or to implement a controller in C++ program would use the software timer on QNX. The test bed provides the basic structure and interface for time scales and networked controllers. For example, researchers could incorporate time scales theory into these models and controllers so that control algorithms could be verified or tailored for efficiency.

1.3 Structure of the Final Report

The structure of the project report is as follows.

Chapter 2 introduces the background and the related work.

Chapter 3 is the project summary, which records what we have done from summer 2010 through summer 2011.

Chapter 4 describes the hardware and software configuration for the test bed, which shows how to setup the system.

Chapter 5 presents the controller design for the rotary inverted pendulum, and the implementation of our simulation environment.

Chapter 6 summarizes the implementation of the C++ controller.

Chapter 7 introduces how to implement a new controller based on the existing example.

Chapter 8 introduces the experiment observations and conclusions.

Appendix A lists the parameters, including their symbols, names and values that are used in defining the pendulum mathematical model.

Appendix B records the hardware and software in current configuration.

Appendix C constitutes the user manual for the test bed.

Appendix D shows how to generate arbitrary time scales using an Agilent 33220A [8] signal generator and software.

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2 Related Work

2.1 Introduction

In this chapter, the related research on networked control system (NCS) and time scales theory is introduced.

2.2 Networked Control System

This section introduces the research topics in NCS.

2.2.1 Overview

A control system aims to regulate the behavior of other systems. Traditional control systems are feedback control systems in which the feedback signals are transmitted through cable or wires. When a traditional feedback control system uses a network to connect the controller to the system, it is called a Networked Control System (NCS). More specifically, an NCS exchanges information (i.e. reference input, plant output, control input) among control system components (sensor, controller, actuator, etc.) using a shared network. A graphical representation of this NCS model is shown in Figure 1 [1].

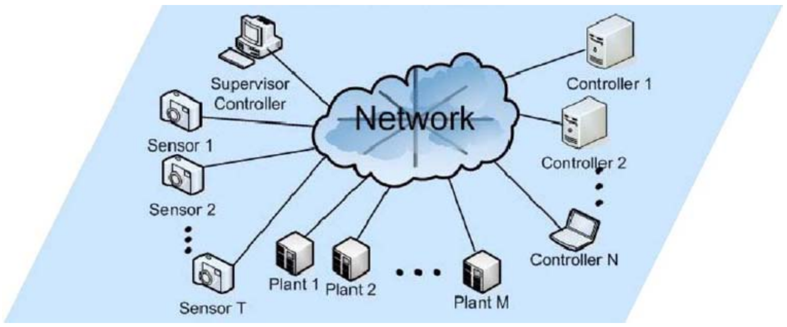

Figure 1 A Conceptual Model of NCS

NCSs have been popular and widely applied for many years because of their numerous advantages, such as reduced wiring, simpler installation, ease of system diagnosis and maintenance, and increased system edibility.

2.2.2 Challenges

There are challenges associated with the design of an NCS. First, there are various kinds of networks, not all of which may be appropriate for a given situation. Choosing an appropriate network may not be trivial. Second, packages transmitted through the network may suffer delay and loss, which influences the performance of the control system. Third, the network may be vulnerable to external attacks. Such a security issue is very important in critical applications.

2.2.2.1 Network Technology Selection

There are many types of networks that the designer can use: Ethernet, CAN, 802.11 a/b/g, bluetooth, ZigBee… Each type of network has its own advantages and disadvantages, so the designer should select the most suitable one for the specific application.

2.2.2.2 Network Delay and Packet Loss Compensation

The network can introduce unreliable/nondeterministic levels of service in terms of delay, jitter, and packet loss. These are problems that are not normally associated with traditional feedback control systems. A common approach is to let the controller solve the problems. Based on the type of delay, there are three solutions [2]: (1) If the delay is constant, the control target can be modeled in a state space equation using the present input and the past inputs. Here, standard Linear Time-Invariant (LTI) control theory can be applied. (2) If delay is variable, and the delay statistics for the network are available, and there is a finite number of delay states, then delay can be treated as a jump-linear system driven by an underlying Markov Chain. (3) If no delay statistics data are available, or if the delay statistics of the states of the Markov chain does not remain constant, then either robust controller techniques or predictor-based methods can be applied.

Usually, the controller applies compensation algorithms to compensate for the delay. Different mathematical-, heuristic-, or statistical-based approaches are used for delay compensation in NCSs, such as the optimal stochastic method, the queuing/buffering method, the robust control method [2-7]. This project does not provide our own delay analysis and delay compensation. However, the test bed has the ability to introduce arbitrary delay and packet loss rate, so that the researchers can investigate their own various algorithms.

2.2.2.3 Network Security Enhancement

Network medium, particularly wireless medium, is susceptible to easy interception. Research in NCS was initiated from the concern for security and convenience in hazardous environments such as nuclear reactor power plants and military applications. In all these applications, security is of the utmost concern. The research in this project does not include NCS security issues from the control system's perspective. The future research should consider these issues.

2.2.3 Improve the System Performance

The delay and packet loss problems can be addressed either within the control system, or within the network system, or in both systems.

The first solution is to address the problem within the control system. Given the network technology, the designer can analyze the statistics of the network. Based on this information, the designer will design the suitable controller that can minimize the influence of the network. The second solution is to improve the network technology to minimize the delay and loss rate, so that the network is compatible for any type of controller. Such methods include: (1) NCS Scheduling approaches and (2) Dynamic bandwidth allocation (DBA) approaches [1].

The above solutions only consider part of the system (either the control system or the network system), which is not complete and efficient. Consequently, some researchers focus on co-design approach, which provides a solution that involves improvement of both the control system and the communication system [8-11]. The key idea of co-design approach is to optimize the control performance by adjusting the control and the network algorithms based on network statistics. For example, A. Chamaken presents a co-design approach for the inverted pendulum system [9]. The authors develop and implement two controllers with control laws considering network statistics. Three different MAC protocols are used in conjunction with two different controllers to stabilize the inverted pendulum. The result shows that the optimal control performance can be achieved at the control layer and the communication layer.

2.3 System Simulation Tools

System simulation tools can be used to simulate the performance of the NCS before the implementation. Simulation is often easier than implementation. Especially when designing the controller, many parameters in control algorithms need fine tunes. In this case, simulation can save time and speed up the process. Since the NCS consists of the control system and network system, first we consider using separate simulation tools for each system. However, this approach is not efficient enough. Hence we decide to use the TrueTime toolbox as the co-simulation tool.

2.3.1 Available Network Simulators

Ns [12] is a discrete event simulator and is widely used in network research. Ns provides a substantial support for the simulation of TCP, routing, and multicast protocols over wired and wireless (local and satellite) networks. It is an open source tool, and is available from http://www.isi.edu/nsnam/ns/.

OMNeT++ [13] is a discrete event simulation environment. Its primary application is the simulation of communication networks. OMNeT++ offers an Eclipse-based IDE and a graphical runtime environment. OMNeT++ is free for academic and non-profit use. It is available from http://www.omnetpp.org/.

OPNET Modeler [14] is a commercial tool developed by OPNET Inc. Modeler incorporates a broad suite of protocols and technologies, and includes a development environment to enable the modeling of all network types and technologies (e.g. VoIP, TCP, OSPFv3, MPLS, IPv6).

A performance comparison of recent network simulators (including ns-2, ns-3, OMNet++) can be found in [11].

2.3.2 Available Control System Simulators

Matlab [15] is a common development environment for a variety of engineering applications. Simulink [15] is an environment for multi-domain simulation and Model-Based Design (MBD) for dynamic and embedded systems. Simulink provides an interactive graphical environment and a customizable set of block libraries that can be used to design, simulate, implement, and test a variety of time-varying systems, including communications, controls, signal processing, video processing, and image processing. Real-Time Workshop (RTW) [16] can generate and execute C and C++ code from Simulink diagrams. The Quanser QUARC library is based on RTW, which can generate and compile C code for various targets. Modelica is a non-proprietary, object-oriented, equation-based language used to model complex physical systems. OPENMODELICA [17] is an open-source Modelica-based modeling and simulation environment intended for industrial and academic usage. Both tools can be used to model a dynamic system. The difference is that Modelica is equation-based so a user must provide explicit ordinary differential equations to represent a system, while Simulink uses “blocks” to represent mathematic operations.

2.3.3 Available Co-Simulation Tools

TrueTime [18] is a Matlab/Simulink-based simulator for real-time control systems. TrueTime facilitates the co-simulation environment of controller task execution in real-time kernels, network transmissions, and continuous plant dynamics.

Ptolemy II [19] is an open-source software framework supporting experimentation with actor-oriented design. The Ptolemy Project has developed directors supporting process networks (PN), discrete-events (DE), dataflow (SDF), synchronous/reactive(SR), rendezvous-based models, 3-D visualization, and continuous-time models.

In this project, we chose TrueTime toolbox working with Simulink as a co-simulation tool. The main reason of the tool selection is that Ptolemy II, with limited available resources, is not that widely used, whereas Matlab/Simulink provides many useful tools for designing control systems, and TrueTime toolbox works perfectly with Simulink to simulate the network system.

2.4 Time Scales Theory

In mathematics, time scale calculus is a unification of the theory of difference equations with that of differential equations, unifying integral and differential calculus with the calculus of finite differences. Consequently, it provides formalism for describing hybrid discrete-continuous dynamical systems [20]. It has applications in any field that requires simultaneous modeling of discrete and continuous data.

One goal of this project is to provide a test bed, including a simulation environment and an experiment system, for researchers to investigate the time scales theory on control systems. The time scales theory is important because conventional control system is classified as either continuous system or discrete system. Time scales theory helps build the connection. However, control theory on time scales has not been much developed. Such a test bed will be useful for future researches.

Billy Jackson [21] examined linear systems theory in the arbitrary time scale setting by considering Laplace transforms stability, controllability, observability, and realizability. Benjamin Allen [22] described the design and implementation of a simulator and real-time controller useful for experimentation with and demonstration of the applications of time scale control theory.

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3 Project Summary

3.1 Introduction

In this project, we developed a test bed to model and simulate embedded control systems. This test bed is designed to

- help users understand digital control systems
- provide a platform for users to define their own controller
- provide a platform for users to simulate their own controller
- provide a platform for users to apply their own controller on hardware
- provide a platform for users to design and verify time scales controllers

This document explains the purpose of the project, what and how things have been done. It is organized in chronological order, summarizing the following semesters: summer 2010, fall 2010 and spring 2011. Each section identifies the objectives and how those objectives were met.

3.1.1 Purpose and Approach

The primary purpose of the project is to develop a test bed that can facilitate researches on time scales theory and networked control theory.

The Baylor University Time Scales Group [1] has done considerable researches on the time scales theory. The theory of time scales was introduced by Stefan Hilger's in his Ph.D. dissertation [2] and subsequent papers as a way to unify the seemingly disparate fields of discrete dynamical systems (i.e. difference equations) and continuous dynamical systems (i.e. differential equations). Time scale systems might best be understood as the continuum bridge between discrete time and continuous time systems.

The test bed allows users to develop and simulate control algorithms, and to generate arbitrary time scales (to regulate sampling rates), so it can be used to implement and verify the time scales theories.

The secondary purpose of this project is to understand digital control systems. Also, this project - the examples and the project report - should be helpful for people who are learning control theory.

We have chosen the rotary inverted pendulum as an example system because it is an unstable system and it is a commonly-used example in the literature. In particular, we use the Quanser SRV02 [3] pendulum system, which consists of a horizontal arm, a vertical pendulum, a gear chain, and a DC servo motor (as shown in Figure 1). The detailed system configuration will be introduced in Chapter 4.

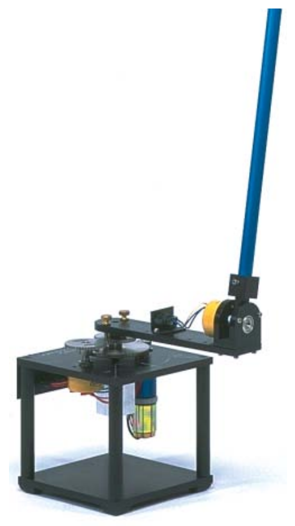


Figure 1 SRV02 Rotary Inverted Pendulum System

3.1.2 Timeline

- This project started during the summer 2010 semester when we set up the hardware and indentified the research problem.
- In the fall 2010 semester, we analyzed the dynamic model, designed and simulated the pendulum controller in Simulink. Also, we re-configured the system with the controller running QNX.
- In the spring 2011 semester, we implemented the controller in Simulink and then in C++.

3.1.3 Accomplishments

We have completed the following tasks:

- Work through Quanser lab 0 [4], lab 1 [5], lab 2 [6], lab 3 [7], lab 7 [8], lab 8 [9]
- Identify relevant research papers
- Understand how to model the pendulum system using Matlab/Simulink

- Understand how to design the controller
- Construct the controller in Simulink and then simulate the system
- Construct the controller in C++

3.2 Summer 2010

3.2.1 Overview

We started the project in the summer of 2010. Dr. Eisenbarth set up the rotary pendulum hardware for the Quanser SRV02, with one PC running Windows XP for both developing and running the controller. Quanser provides eight labs that demonstrate how to utilize the hardware and software. I worked on doing these labs, as well as indentifying the research problems.

3.2.2 Objectives

Our objectives during that time were to

- Understand how to use the Quanser software and hardware
- Carry out the rotary pendulum experiments provided by Quanser
- Investigate SysML [10] modeling
- Investigate Modelica [11] modeling and simulation
- Investigate model transformation using VIATRA [12]
- Read papers on network control system and identify research problems

3.2.3 What Did I Do?

3.2.3.1 Quanser Labs

On the software side, I followed the Quanser manual [3] and did Lab 0-3, and 7-8. This helped me understand how to install Quanser software, how to create a Quanser Simulink model, how to configure the target machine, and how to compile and run a model.

On the hardware side, I worked with Dr. Eisenbarth to setup the hardware. Initially, we had only one Windows XP PC used for both developing and running the controller. However, we found that sometimes the pendulum fell down after a short time. Probably it is because that Windows XP is not a real time operating system. Therefore, Dr. Eisenbarth added a second PC running the QNX Neutrino [13] real time operating system. This is the current system configuration, in which the QNX computer is only for running the controller and the Windows XP computer is only for developing the control algorithms. The design process is that the controller is first simulated on Windows XP, and then compiled, downloaded and run on the QNX computer. Since this configuration is setup, the pendulum does not fall down again.

3.2.3.2 SysML modeling

We planned to use SysML to describe the system, so I read the book *Systems Engineering with SysML/UML* [14] (particularly Chapter 4) which describes the basic concepts and diagrams in SysML.

The Systems Modeling Language (SysML) is a general purpose modeling language for system engineering applications. It supports the specification, analysis, design, verification and validation of a broad range of systems and systems-of-systems. Such systems may include hardware, software, information, processes, personnel, and facilities [10]. Here SysML can be used to describe the structure of a control system.

Artisan Studio [15] is a tool that supports SysML modeling and I became familiar with it by completing one of its tutorials [16]. I constructed a SysML model to represent the water distiller example from Chapter 15 of [14]. Also, I constructed a SysML model of the pendulum system, including Block Definition Diagram (BDD), Internal Block Diagram (IBD), activities diagrams and state charts.

3.2.3.3 Modelica and Simulink modeling

We investigated the possibility to use Modelica (in conjunction with SysML) or Simulink to model the system.

Modelica [17] is a non-proprietary, object-oriented, equation-based language used to model complex physical systems. OpenModelica [11] is an open source Modelica simulation environment. I followed the OpenModelica Users Guide [18] to learn how to simulate a system, and I replicated the tank example in Chapter 15 of [19].

Simulink [20] is an environment for multi-domain simulation and model-based design for dynamic and embedded systems. It provides an interactive graphical environment and a customizable set of block libraries that enable users to design, simulate, implement, and test various applications, including communications, controls, signal processing, video processing, and image processing.

Both tools can be used to model a dynamic system. The difference is that Modelica is equation based so that a user must provide explicit ordinary differential equations to represent a system. On the contrary, Simulink uses blocks to represents mathematic operations.

3.2.3.4 VIATRA2

Model transformation is important in model-driven engineering. In this project, model transformation is used to transfer the Simulink model into an executable binary code. So how to do model transformation is one of the research problems to be investigated.

VIATRA2 (VIsual Automated model TRAnsformations) [12] framework provides a general-purpose support for the entire life-cycle of engineering model transformations. In particular, it provides a means to uniformly represent models and metamodels, and a high performance transformation engine. To understand the framework, I read the *VIATRA 2 Model Transformation Framework User's Guide* [21], Chapter 1-3. And I did VIATRA2 *hello world* and *Transforming UML activity diagram into a Petri net* tutorial. But at current stage, model transformation has not been done yet. The VIATRA2 tool has not been used.

3.2.3.5 Other relevant research

Thomas Johnson's thesis, *Integrating Models and Simulations of Continuous Dynamic System Behavior into SysML* [22], describes how SysML and Modelica can be used in concert. The objective of his research was to use graph patterns and transformation rules to integrate models of continuous dynamic system behaviors (represented using Modelica) with SysML models. This would provide a more comprehensive modeling approach.

I used IEEExplore to locate papers relevant to my project based on the keyword "Networked Control System" (NCS). I selected about 20 papers, read through each one, and selected the interesting ones and added them to my RefWorks repository [23-28]. In particular, I found that NCS co-design, which considers the characteristics of both the network and control system and solves the problem as an integrated system, is very interesting [29-33] and focused on it during the fall 2010 semester.

3.3 Fall Semester 2010

3.3.1 Overview

During the fall 2010 semester, I set up the rotary pendulum simulation environment in Simulink and developed the controller. The simulation results demonstrated that the controller could successfully balance the pendulum. I also investigated how to simulate an Network Control System (NCS). After investigating several different tools [11, 20, 34-39], I finally chose the TrueTime toolbox [35].

3.3.2 Objectives

The objectives of the fall semester were to

- Understand how to model the dynamics of the rotary pendulum
- Understand the Quanser Simulink model
- Implement the Simulink model that simulates the balance controller
- Investigate networked control systems
- Implement an NCS controller for the rotary pendulum
- Determine how to use the TrueTime toolbox to simulate wireless networks

3.3.3 What Did I Do?

3.3.3.1 Investigate NCS simulation tools

I searched the Internet for network and control systems simulation tools and found several relevant types: co-simulation tools (e.g., TrueTime [35], Ptolemy II [38], PiccSIM [39]), network simulators (e.g., OPNET [37], NS-2 [34], NS-3 [34], OMNeT++ [36]), and control system simulation tools (e.g., Simulink [20] and Modelica [11]).

I evaluated these tools, based on their capabilities: the related programming languages, the operating system that is assumed, the compatibilities of working with other tools and required resources. After evaluating each tool, I found that the TrueTime toolbox would work well for NCS co-simulation, because the TrueTime toolbox provides Simulink blocks that are targeted for network simulation, making it easy to set up an NCS environment in Simulink.

3.3.3.2 Practice Simulink modeling

I replicated the tank example from page 385 of *Principles of Object-Oriented Modeling and Simulation with Modelica 2.1* [19]. The original model is represented in Modelica, and I created an equivalent model in Simulink. Figure 2 shows the structure of the tank system. Liquid enters the tank through a pipe from a source (on the left), and leaves the tank via another pipe (on the right) at a rate controlled by a valve. The liquid level in the tank must be maintained at a fixed level as closely as possible, no matter what the input flow is.

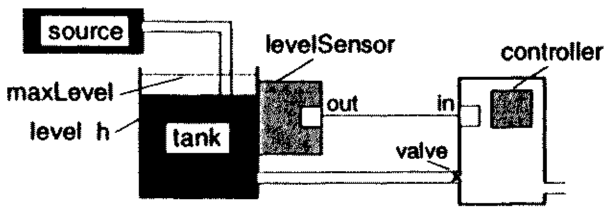


Figure 2 A Tank System

```
model Tank
   ReadSignal tSensor   "Connector, sensor reading tank level (m)";
   ActSignal  tActuator "Connector, actuator controlling input flow";
   LiquidFlow qIn       "Connector, flow (m3/s) through input valve";
   LiquidFlow qOut     "Connector, flow (m3/s) through output valuve";
   parameter  Real area(unit = "m2")       = 0.5;
   parameter  Real flowGain(unit = "m2/s") = 0.05;
   parameter  Real minV = 0, maxV = 10;// Limits for output valve flow
   Real       h(start = 0.0, unit = "m") "Tank level";

   equation
      assert(minV >= 0, "minV - minimum Valve level must be >= 0");
      der(h) = (qIn.lflow - qOut.lflow)/area;  // Mass balance

   equation
      qOut.lflow = LimitValue(minV, maxV, -flowGain*tActuator.act);
      tSensor.val = h;
   end Tank;
```

Figure 3 A Modelica Model

The Modelica source code for the tank is shown in Figure 3. Note that each physical object is represented as an object (or variable) and equations represent continuous behaviors.

The out flow valve is controlled by a PI controller to keep the height of the liquid at 0.25 meters, where the parameters for the PI controller are P = 2 and I = 0.1. The control system diagram is shown in Figure 4.

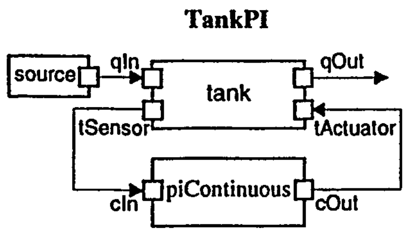
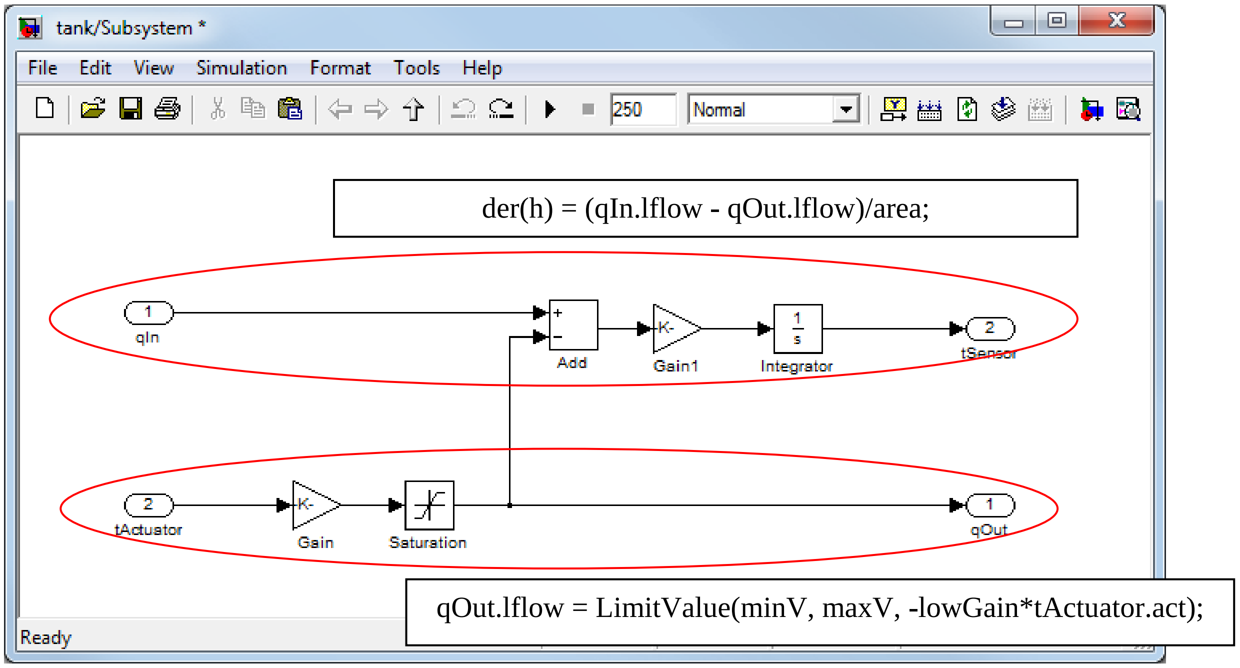


Figure 4 A Tank Simulink Model with a Continuous PI Controller

To model the system, I first created the tank subsystem. Here I replaced the ordinary differential equations in Modelica with Simulink blocks. The corresponding equation and blocks are marked in Figure 5 , with the associated equations shown in the figure.



der(h) = (qIn.lflow - qOut.lflow)/area;

qOut.lflow = LimitValue(minV, maxV, -lowGain*tActuator.act);

Figure 5 Simulink Subsystem

A continuous PI controller was designed to control the liquid level in the tank. Figure 6 shows the Simulink control system diagram. The Subsystem in Figure 6 is connected to the PI controller to form a closed-loop system.

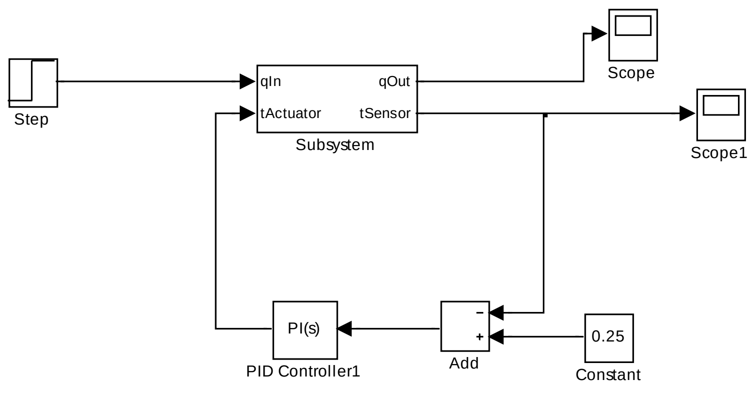


Figure 6 Simulink Model of the Local Control System

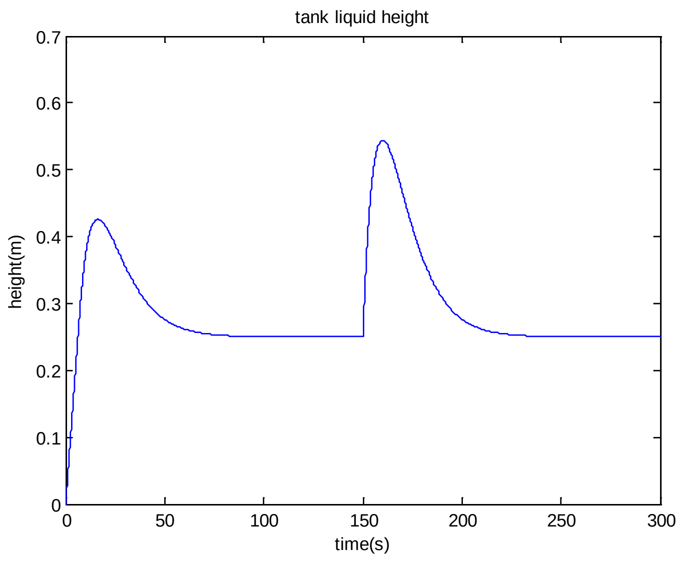


Figure 7 Simulation Result of the Local Control System

To simulate the tank system, we assume that there is an input liquid flow, which is the input signal (i.e., qin). Assume that the input flow maintains at 0.02 m^3/s from 0 to 150 second, and then increases to 0.06 m^3/s after 150 second. The height of liquid in the tank is shown in Figure 7. At t = 0, the height is 0, and the liquid begins to flow into the tank. After the height is over 0.25 m, the PI controller adjusts the valve to maintain the height at 0.25 m. At t = 150, the input liquid flow increases, so here is an overshoot in the height. But the PI controller maintains the liquid level to 0.25 m.

After successfully creating the Simulink model and running the simulation, I embedded a TrueTime network block in the model and completed an NCS, as shown in Figure 8. The upper collection of blocks represents the tank, sensor and actuator system, and the lower collection of blocks represents the controller system. These two collections communicate through TrueTime send and receive blocks, which are configured to use the Ethernet protocol. The tank system uses a Sensor Trigger to sample the liquid height, and sends the liquid height information through the TrueTime (Sensor) block. On the controller side, the TrueTime Receiver (Controller) block receives the data, calculates the control signal, and sends it through the TrueTime Send (Controller) block. The TrueTime Receive (Actuator) block receives the control signal, and adjusts the actuator accordingly. The simulation result is shown in Figure 9, which is identical to the simulation result of local control system in Figure 7.

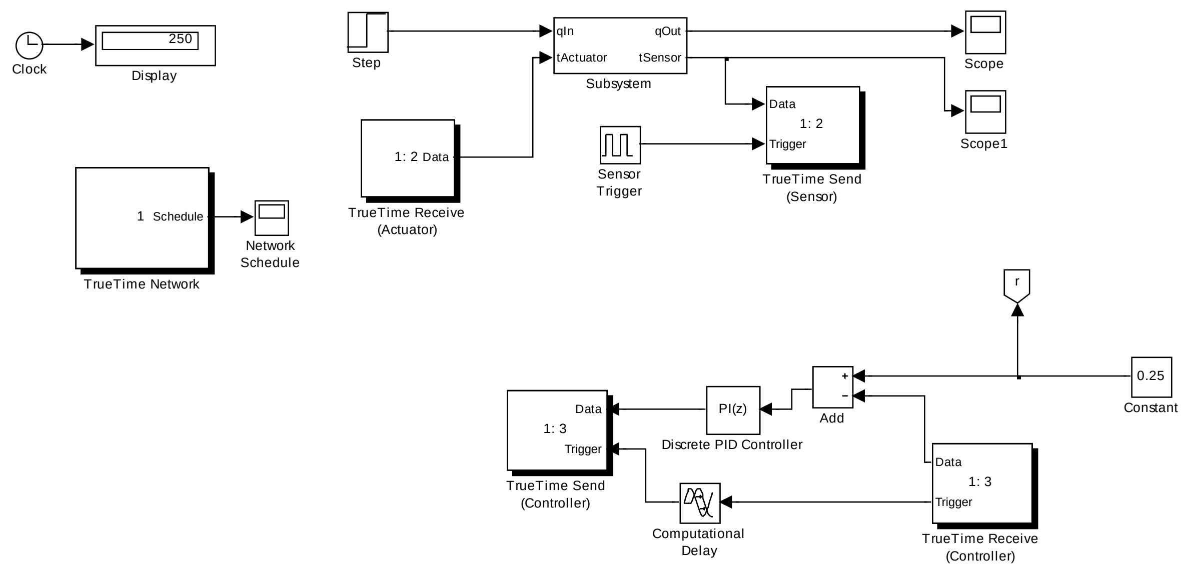


Figure 8 Simulink Model of the Networked Control System

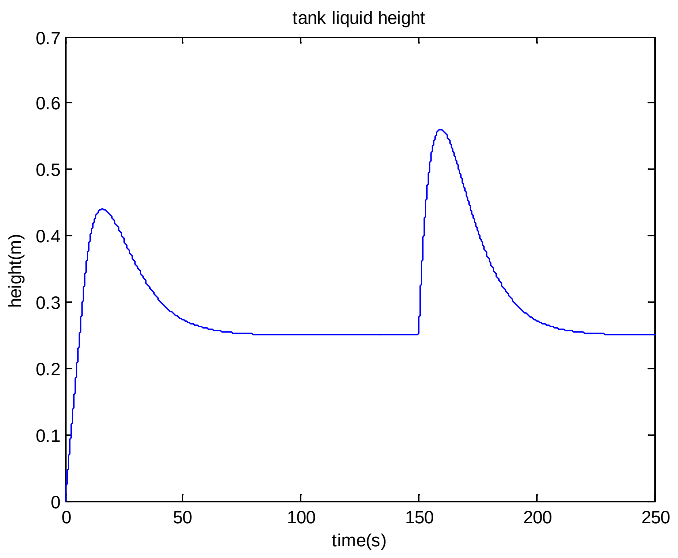


Figure 9 Simulation Result of the Networked Control System

3.3.3.3 Model Rotary pendulum and Design Controller

At the beginning of the fall semester, I read the Quanser manual [9] to understand how to model the pendulum dynamics. The dynamic equations can be constructed by using the Lagrangian [40]. I created a non-linear Simulink model (Figure 11) based on the dynamic equations to represent the pendulum system. The pendulum system is a non-linear system. But in order to simplify the problem, we linearize the equations by assuming that the pendulum angle is close to 0 when balanced. This results in a state space representation, which is a linear system – an approximation of the actual non-linear system.

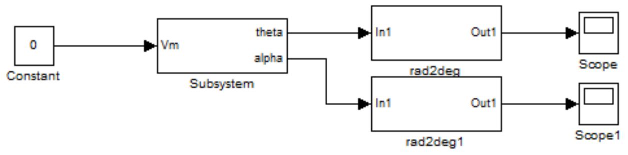


Figure 10 Simulink Rotary Pendulum Model

The performance of the linear approximation can be analyzed by comparing it with the respective non-linear Simulink model. I created the first Simulink model (Figure 10) that utilizes the non-linear model. Initially, the simulation initial condition sets the pendulum angle to 0.01 rad, which is a small non-zero value representing an unbalanced condition. The simulation sets the input to 0 volts, simulating the free-falling nature of the pendulum. Similarly, I created the second Simulink model with the non-linear subsystem replaced by a state space subsystem. After running the simulations, I compared the outputs (arm position and pendulum position) of the two models. The result should indicate that the output of the linear model matches the non-linear model when the pendulum angle is small, and the output of the linear model deviates from the non-linear model when the pendulum angle becomes large. However, my non-linear block simulation result showed an unchanged pendulum angle, which was incorrect. Since the non-linear model was not used in the project, I did not investigate this issue further.

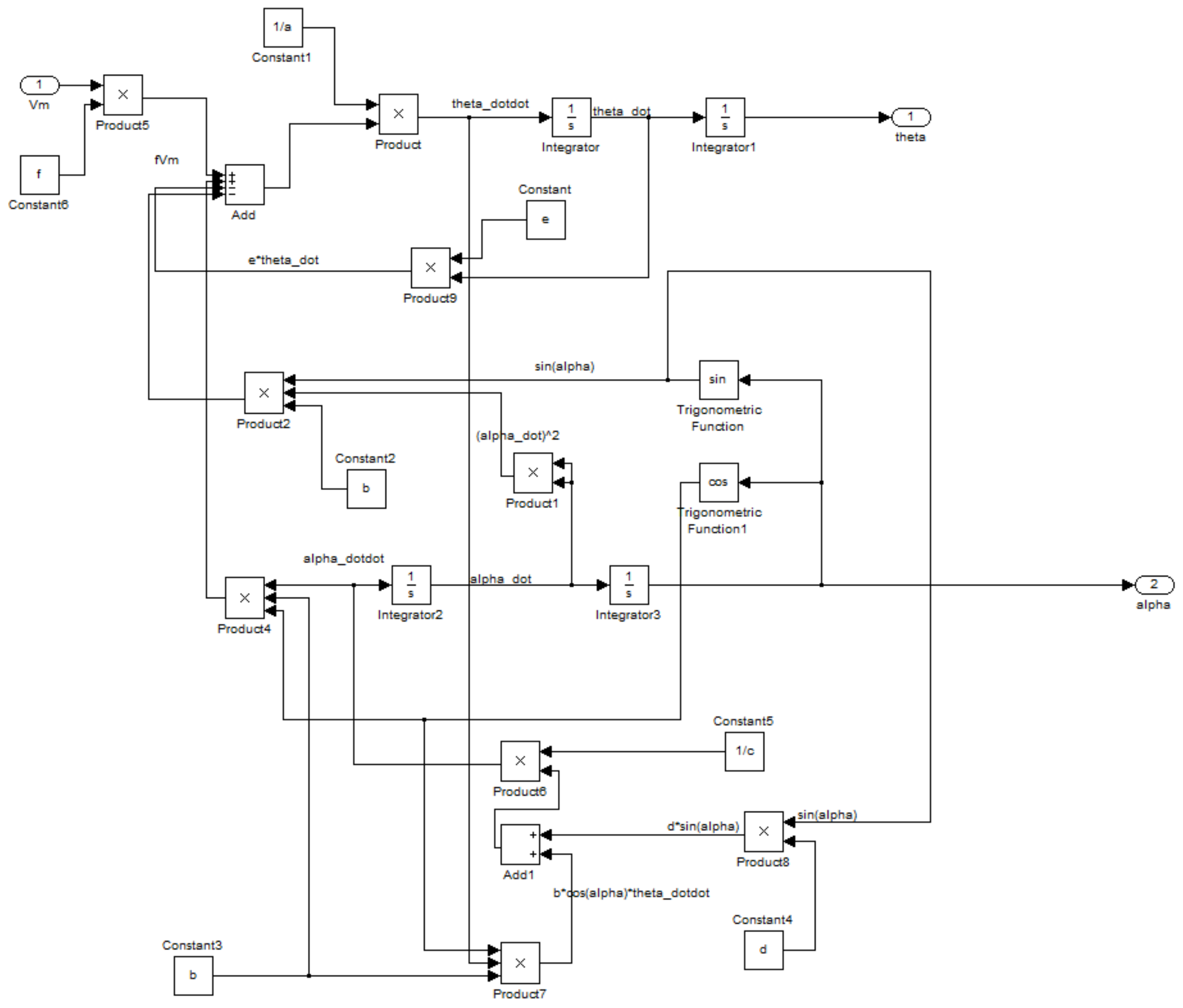


Figure 11 Non-linear Pendulum Model

3.3.3.4 Design the Controller

During the fall 2010 semester, I took the control theory class, ECL 4337, which focused on the design of continuous control systems, using the textbook *Modern Control Systems* [41]. Since this project requires knowledge of digital control systems, I referenced the book *Digital Control of Dynamic Systems* [42], which describes how to design a digital feedback control system including a controller and an observer.

3.3.3.5 Implement Simulink Controller Simulations

I implemented the following simulation models in Simulink.

- Simulation with the motor dynamics using a filter to estimate the velocity (Figure 12)
- Simulation with the motor dynamics using a state space observer to estimate the state variable **x** (Figure 13)
- NCS simulation (Figure 14)

For the rotary pendulum system, the state variable **x** is defined as

$$\mathbf{x} = \begin{pmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{pmatrix}$$

where θ is the pendulum position, $\dot{\theta}$ is the pendulum velocity, α is the horizontal arm position, $\dot{\alpha}$ is the arm velocity.

The state-space representation of the inverted pendulum is

$$\begin{gathered} \dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}u \\ \mathbf{y}=\mathbf{C}\mathbf{x}+\mathbf{D}u \end{gathered}$$

where u is the voltage applied to the DC motor. The Simulink model for the pendulum system is shown in Figure 12 and Figure 13. The details of controller design will be introduced in Chapter 5.

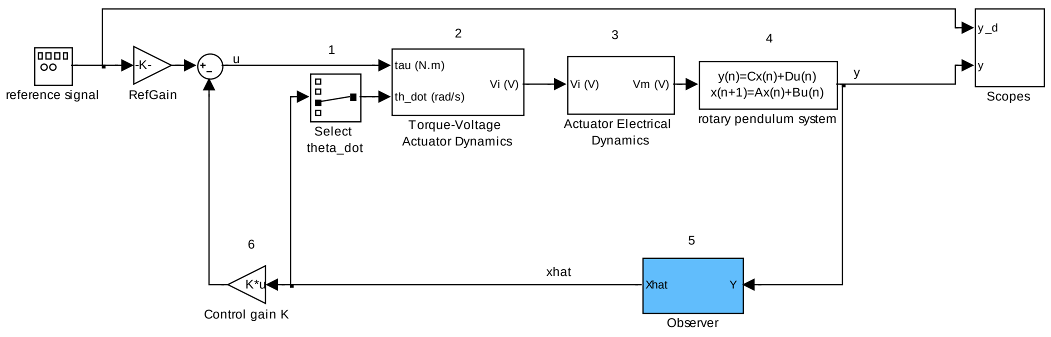


Figure 12 Simulink Model with a Filter Observer

For our system, however, the encoder only has access to θ and α , and is missing $\dot{\theta}$ and $\dot{\alpha}$, so an observer is needed to estimate the velocities, $\dot{\theta}$ and $\dot{\alpha}$. The observer can be implemented in two ways. The observer (subsystem 5) in Figure 12 utilizes a differentiator and a filter to estimate the velocities, while the observer (subsystem 5) in Figure 13 uses a state space observer to estimate the velocities. The filter observer is specific to this application, but I need to investigate more general cases. Therefore, I decided to use the state space observer.

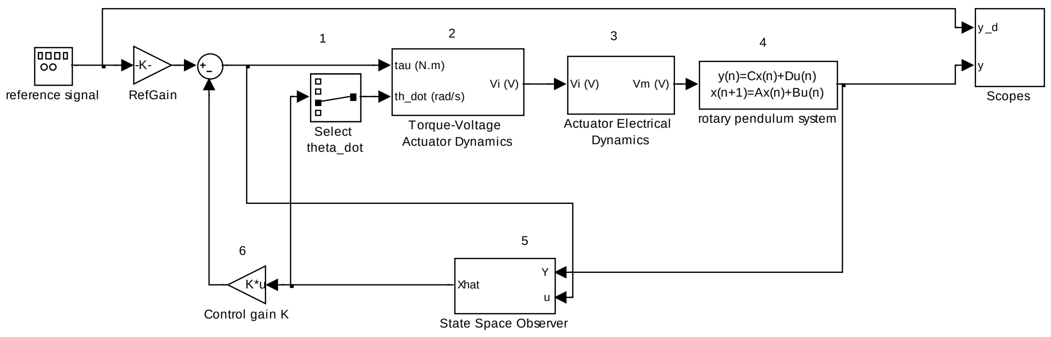


Figure 13 Simulink Model with a State-Space Observer

The above models (Figure 12 and Figure 13) represent local control systems. To simulate a networked control system and investigate its performance, I created an NCS model (Figure 14) using a TrueTime Network block, which can be configured to be one of several networks (e.g. Ethernet, CAN), and various parameters, such as bit rate, loss rate. In Figure 14, subsystems 1-6 are the same as the local control system (Figure 12 and Figure 13). But the system now is divided into two isolated systems: the controller system and the pendulum system. In the pendulum system (blocks 1-4, 7, 10), the sensors (block 7) get the position information and sends it to the controller, where the data rate is controlled by the sensor trigger. The pendulum receiver (block 10) receives the control signal, and writes it to the actuator. In the controller system (subsystems 5, 8, 9), the controller receiver (block 8) receives the position information, calculates the control signal, and sends it back to the actuator through controller sender (block 9).

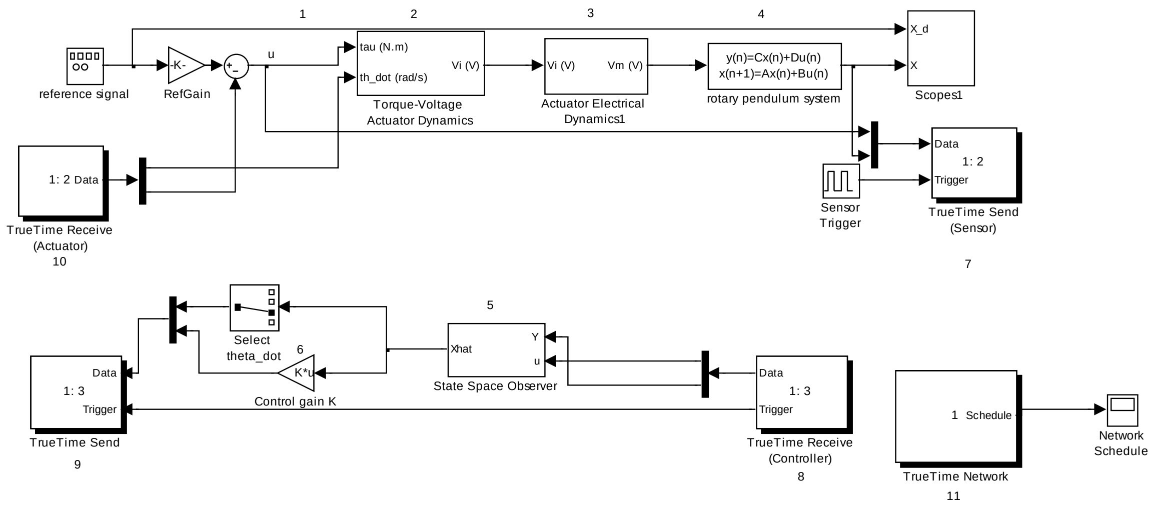


Figure 14 Simulink model of NCS

3.4 Spring Semester 2011

3.4.1 Overview

During the spring 2011 semester, I implemented the controller that I had simulated during the previous semester. The first step was to implement it in Simulink. After I got the controller working, I found that Simulink had the limitation that the sampling rate could not be varied. This restriction had to be overcome because we wanted to use the time scales theory, which would require the sampling rate to be altered during runtime. Dr. Eisenbarth suggested that we attach a signal generator (i.e. Agilent 33220A) to the external interrupt pin on the Q4 board. We could then use the Agilent software to generate arbitrary time scales. This solution allowed us to predefine several time scales, which could then be chosen at runtime.

However, the time scales could not be changed after it is downloaded to the signal generator. So I implemented a software solution, which was a C++ representation of the controller running on QNX. QNX provides software timers that can be used in the C++ project. When a timer expires, the program directly accesses the Q4 board and calculates the control signal. So we can set the timer expiration time at each step, which gives us the flexibility to change the sampling rate.

During this time, to understand time scales theory on control systems, I read Benjamin Allen's thesis, *Experimental investigation of a time scales linear feedback control theorem* [43], and Billy Jackson's dissertation, *A General Linear Systems Theory on Time Scales: Transforms, Stability, and Control* [44].

3.4.2 Objectives

The objectives of this semester were to

- Implement my controller in Simulink
- Implement my controller in C++
- Study the time scales theory
- Investigate the stability of switched systems

3.4.3 What Did I Do?

3.4.3.1 Implement the balance controller in Simulink

I implemented an executable model in Simulink, which can be compiled, downloaded and run on QNX. However, when I first implemented the controller the controller did not balance the pendulum. It seemed that the motor did not output enough torque. After investigating the Quanser Lab 8 and each sub-system in the Quanser example, I found that I had failed to include the motor dynamic sub-system in my model. After adding the motor dynamics sub-system, which converts the torque to a voltage signal that is applied to the motor, the controller can balance the pendulum.

3.4.3.2 Read Ben's thesis and Jackson's dissertation on time scales theory

I read Benjamin Allen's thesis, *Experimental investigation of a time scales linear feedback control theorem* [43], and Billy Jackson's dissertation, *A General Linear Systems Theory on Time Scales: Transforms, Stability, and Control* [44]. Based on Ben's Matlab code, I implemented a Matlab program to simulate the time scales controller. However, there were two problems when we considered the time scales control.

(1) How to design the observer. Currently time scales control theory can only solve full-state feedback control problem. In the pendulum system, we only have position information, but no velocity information. So our system has only partial state information. Also, the conventional observer technique generally does not work when using time scales. In my simulation, I calculated the observer matrices at each step, using the conventional pole placement algorithm. The simulation result showed that the pendulum would converge only after a long time. At this point we lack the theory so this problem remains unsolved.

(2) How to vary the sampling rate while a model is running. To use time scales control, we need to vary the sampling rate. In Simulink, the sampling rate is a system parameter that should be set before simulation. Once it is set, we cannot change it. Besides, when compiled into executable code, the Quanser software creates a thread with fixed scheduling period on QNX. Therefore, it is difficult to generate and use time scales in Simulink.

3.4.3.3 Use external signal generator to generator time scales

To solve Problem Two, we attached a signal generator directly to the Q4 extension board to control the sampling rate via the external interrupt pin on the Q4 extension board. To utilize this signal, the sensor reading block must be configured to use an external interrupt as the clock. After these hardware changes were made, we did experiments to determine whether the Simulink model's sampling rate was in fact controlled by the external interrupt signal, and the answer was yes.

3.4.3.4 Implement the balance controller on QNX using C+

Though the time scales can be generated by external signal generator, we could not directly control the signal generator from the Simulink model. Therefore, I solved Problem Two by implementing a C++ version of the controller mimicking the Simulink model, where the time scales was generated by software timer. I developed the project using QNX Momentics IDE. To achieve a varying sampling rate, I use a software timer to control the sampling rate. The QNX provides software timers that can be set. When the timer expires, the OS sends a signal to the controller program. Upon receiving the signal, the program directly accesses the Q4 board to read sensor data and calculates the control signal. We can set the timer expiration time after it expires so that the sampling rate can vary. This gives us the flexibility to generate time scales while the controller is running. At the beginning, I had some difficulty compiling the project. But after I set the correct library/header path, and linked to the correct Quanser library, the compilation was successful.

3.4.3.5 Investigate the switch system stability

One of our objectives is to investigate the stability of the system when it switches from a stable state to an unstable state, and vice versa. From time scale theory, we know that there exists a $\mu_{\max}$ and corresponding Hilger circle associated with the control system. $\mu_{\max}$ is the largest sampling period that a digital controller can use and still guarantee stability. Assume that the system's graininess is μ, the

Hilger circle has a diameter equal to the reciprocal of graininess and is tangent to the imaginary axis [45]. If the system poles stay within the Hilger circle, the system will remain stable. As the sampling period (μ) varies, the Hilger circle changes. The system poles may go outside the Hilger circle if μ is too large. However, there is a conjecture that as long as the system does not stay outside the Hilger circle "too long", the system is still stable. To demonstrate this, I simulated what happened when the system switched forth and back inside and outside of the Hilger circle.

First, I used the Simulink model to find the sampling rates (1) that would balance and (2) that would not balance the pendulum. Let Ts represents the sampling period. By doing several experiments, I found that when Ts <= 10 ms, the pendulum is stable, and when Ts = 20 ms the pendulum is unstable.

Second, I investigated the case where the control system switched between the stable and unstable case. For the simulation, I created a Matlab program to model the switching mechanism, where the switch criterion was based on the norm of difference between the reference vector and the state variable.

$$T_s = \begin{cases} 5\text{ms} & \text{if } \|\mathbf{ref} - \mathbf{x}\| > 0.04 \\ 20\text{ms} & \text{if } \|\mathbf{ref} - \mathbf{x}\| \leq 0.04 \end{cases}$$

Figure 15 shows the position and Ts when the horizontal arm is tracking a square-wave reference input with amplitude 0.1 rad. Here, the above figure shows the arm position and pendulum position. We can see that the arm can track the reference position, and the pendulum angle is within 0.04 rad, indicating that the pendulum is balanced. The figure below shows the switching sampling period, where Ts switches between 5ms and 20ms. When the reference input remains unchanged, the system is sampled at 20ms. However, when the reference input changes, system must be sampled at 5ms.

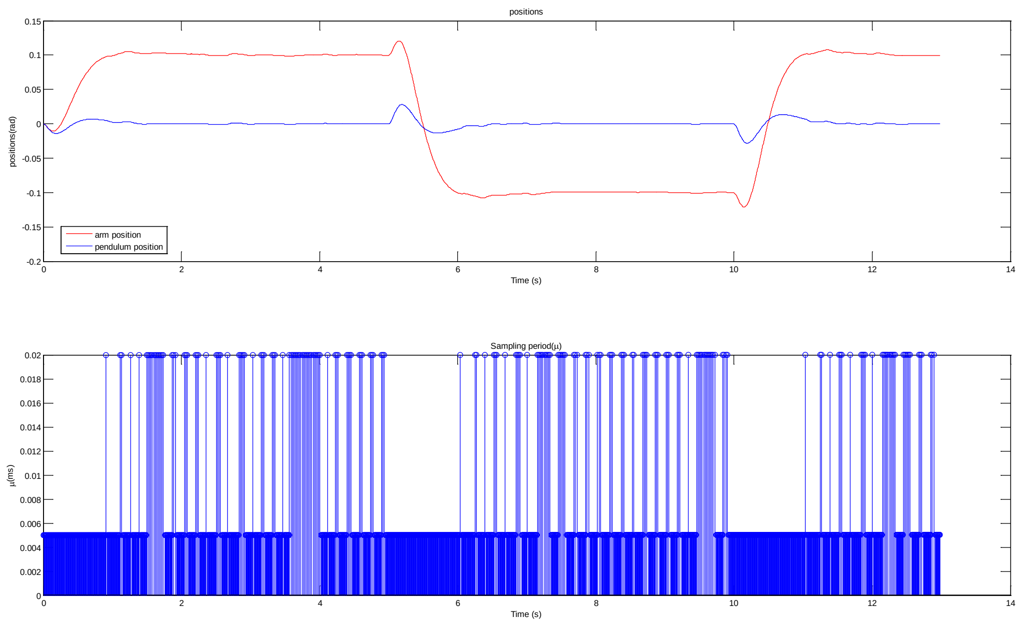


Figure 15 Simulation Result of the Switched System

Then I implemented and executed this switched system in C++ project, which was consistent with the simulation.

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4 The System Configuration

The hardware and software that we have used in the project are described in this chapter. For a detailed list of the hardware and software (plus contact information), please refer to Appendix B.

4.1 Hardware

The hardware includes the following components: a host computer (Host) running Windows 7 (on which a virtual machine was installed to run Windows XP), a target computer (Target) running QNX Neutrino Realtime Operating System, a Quanser Q4 data acquisition board and a terminal board, a UPM-1503 Power Module, and a SRV02-E servo plant and pendulum (Rotary Pendulum).

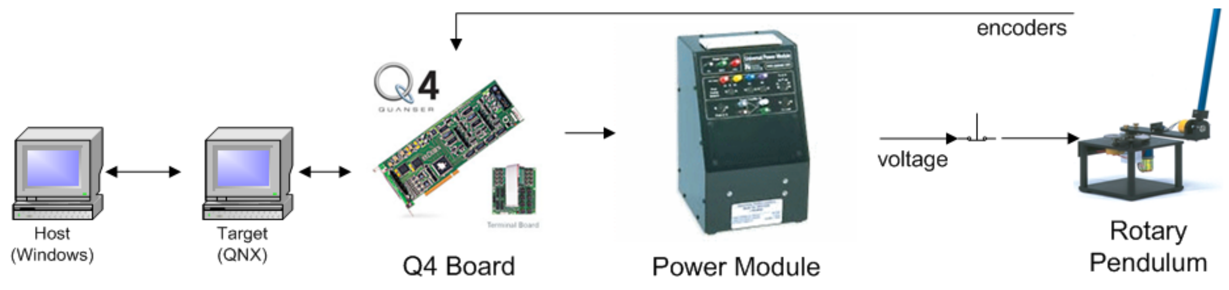


Figure 1 Hardware Configuration

4.1.1 Computers

The Host is used to develop the control algorithm and to run a simulation, using the Matlab/Simulink environment. By utilizing the Quanser software, a Simulink model can be compiled on the Host computer, and the executable can be downloaded and run on the Target. The Target runs the QNX realtime operating system and is used to execute the controller.

4.1.2 SRV02-E Servo Plant

The Rotary Pendulum module is from Quanser Corporation. It consists of a horizontal arm and a vertical pendulum attached to a Quanser SRV02-E servo plant (as shown in Figure 2).

The Quanser SRV02-E servo plant is the base of the pendulum system. It has a metal frame that houses a gear drive connected to a DC servo motor. The horizontal arm, which rotates in the horizontal plane, is mounted on the outside gear of the gear drive. The gear drive is driven by the DC motor and has two configuration options: low-gear configuration (left in Figure 3) and high-gear configuration (right in Figure 3). This project used the high-gear configuration.

The SRV02-E is equipped with two optical incremental digital encoders that measure the horizontal arm position and the pendulum position, respectively. The encoders provide a high resolution, i.e. 4096 counts per revolution, in quadrature mode. The encoder attached to the arm shaft measures the horizontal angle of the arm, defined as θ. The other encoder, attached to the pendulum hinge, measures the vertical angle of the pendulum, defined as α (shown in Figure 2). Both α and θ are measured in radians.

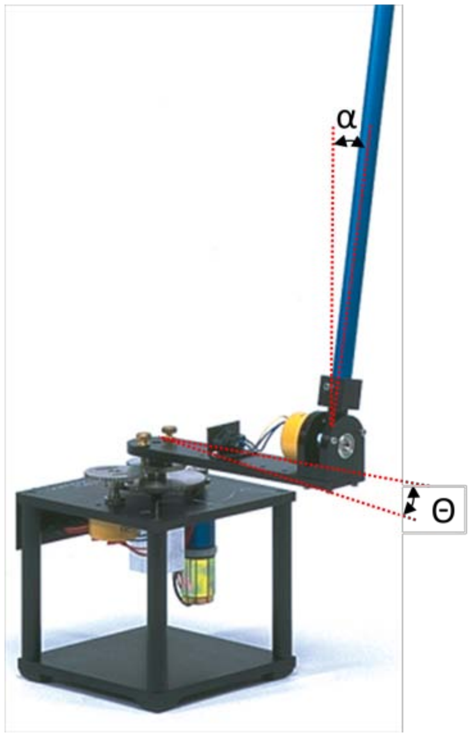


Figure 2 Rotary Inverted Pendulum

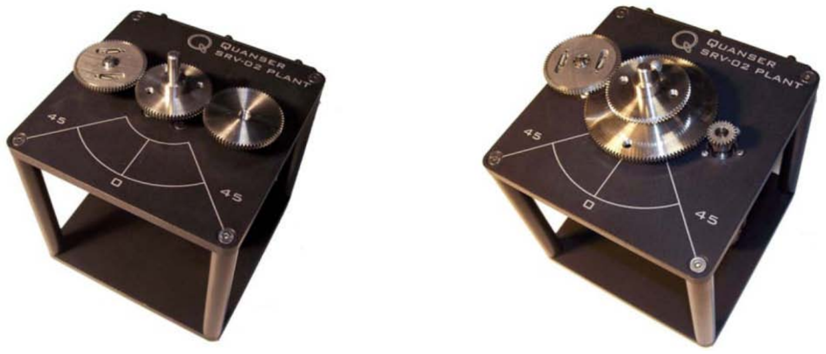


Figure 3 Gear Configuration

4.1.3 UPM-1503 Power Module

The UPM-1503 (Universal Power Module) contains a ±12 volt power supply, analog sensor input and power amplified analog output. It is used to drive the DC motor.

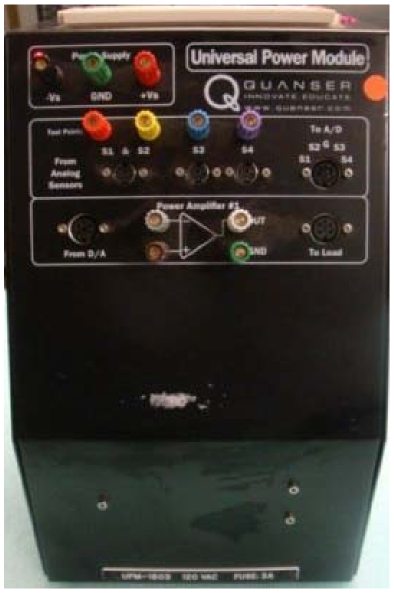


Figure 4 UPM-1503

4.1.4 Quanser Q4 board

The Quanser Q4 board is really two boards: a Data Acquisition (DAQ) board (shown in Figure 5) and an external terminal board (shown in Figure 6). These boards connect the controller to the pendulum system. The DAQ board is installed on the Target’s PCI bus, and is connected to the external terminal board via a flat cable. The terminal board provides ports for digital encoder input, analog input, analog output, and digital input. For this project, we have only used the digital encoder input and the analog input ports.

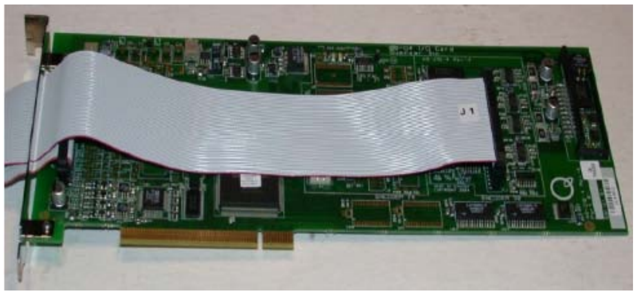


Figure 5 Quanser Q4 Data Acquisition Board

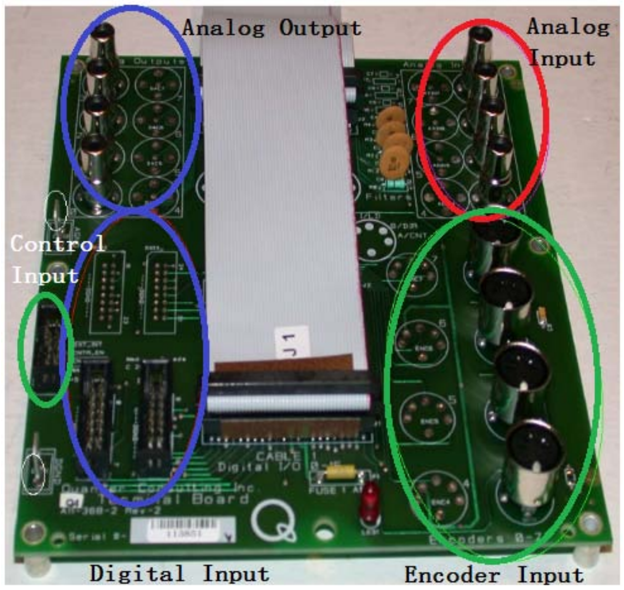


Figure 6 Quanser Q4 Terminal Board

4.2 Software

4.2.1 Quanser Library

Quanser QUARC library allows control prototyping and hardware-in-the-loop testing. It is integrated with Simulink and the Real Time Workshop (RTW). The QUARC library extends the code generation capabilities of RTW by adding a new set of Targets, such as Windows and QNX x86. The Target OS setting determines the code generated by the RTW. This allows the user to compile the C source code generated from the model, to link it with the appropriate libraries for the particular Target platform, and to download the code to the Target via an Ethernet connection between the Host and the Target.

The QUARC library supports data acquisition cards from other manufacturers, such as National Instruments. The library also provides a driver for the Q4 board so that the controller can use high-level functions from the QUARC library to complete low-level IO tasks.

4.2.2 Matlab and Simulink

We have used Matlab as our basic development platform because the Quanser hardware and software works seamlessly with Matlab and Simulink. Matlab is a textual programming language, whereas Simulink is a graphical programming language that allows the user to define a system as a collection of interconnected “building blocks”, where the blocks are manipulated graphically.

In this project, we developed our own Simulink version of a controller, which then served as the architecture for a controller that we implemented in C++ (executed without using Matlab or Simulink).

4.2.3 QNX

QNX is a realtime operating system (RTOS) from QNX Software Systems. We are currently running the QNX Neutrino RTOS. A RTOS is necessary here because the timing constraints are critical.

The C++ development system for QNX is their QNX Momentics Tool Suite, an Eclipse-based integrated development environment. In addition to its editing feature, the tool suite can give developers an at-a-glance view of realtime interactions and memory profiles.

5 Pendulum Controller Design

This chapter introduces how to design a controller to balance the rotary inverted pendulum.

5.1 Physical model

The rotary pendulum is an unstable system that requires a controller to keep the pendulum at its upright position. The controller incorporates a physical model for the pendulum, i.e. mathematical equations that characterizing its structure and behavior. This section describes those equations.

5.1.1 Non-Linear Equations of Motion

The simplified diagram of the rotary inverted pendulum is shown in Figure 1. The polar coordinate system is used to describe the inverted pendulum system, where α is polar angle of the vertical pendulum and θ is the polar angle of the horizontal arm with the directions shown in Figure 1. These angles are measured by the encoders, which increases when the arm/pendulum rotates Counter Clockwise (CCW). The physical model can be analyzed by using the Lagrangrian. The nonlinear equations of motions are [1]

$$\begin{gathered} 2m_p l_p^2 \dot{\alpha}\dot{\theta}\sin\alpha\cos\alpha - m_p l_p r\sin\alpha(\dot{\alpha})^2 + (m_p r^2 + m_p l_p^2 - m_p l_p^2(\cos\alpha)^2 + J_{arm})\ddot{\theta} \\ + m_p l_p r\ddot{\alpha}\cos\alpha = \tau_m - B_{arm}\dot{\theta} \end{gathered} \tag{1}$$

and

$$-m_p l_p^2(\dot{\theta})^2\sin\alpha\cos\alpha + m_p l_p r\ddot{\theta}\cos\alpha + (J_p + m_p l_p^2)\ddot{\alpha} + m_p g l_p\sin\alpha = B_p\dot{\alpha} \tag{2}$$

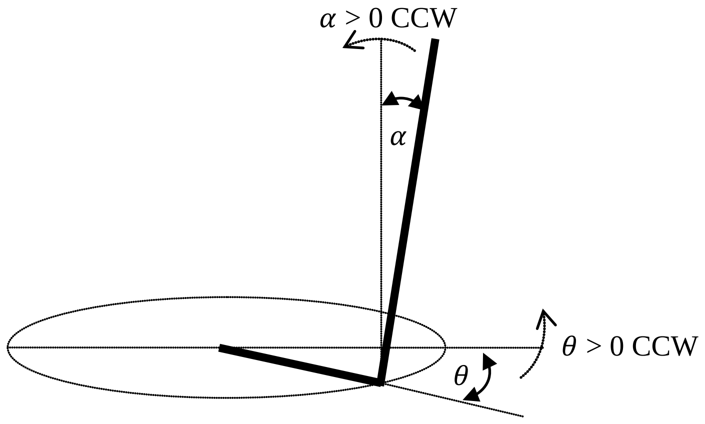


Figure 1 Rotary Inverted Pendulum

where m_p is the pendulum mass, l_p is the distance from pendulum pivot to centre of gravity, r is Full Length of the pendulum, J_{arm} is moment of inertia acting seen from arm pivot, τ_m is the torque applied at the load gear. For a complete listing of the symbols used in the equations (1) and (2), please refer to Appendix A.

In (1), τ_m, with unit Nm, is the torque applied to the gear. It is generated by the servo motor. Given the torque, it is necessary to convert the torque to a voltage that will be applied to the DC motor. By analyzing the motor dynamics [1], we have

$$\tau_m = \frac{\eta_g \eta_m K_g K_t (V_m - K_g K_m \dot{\theta})}{R_m} \tag{3}$$

where η_g is gearbox efficiency, η_m is motor efficiency, K_g is total gearbox ratio, K_t is motor torque constant, K_m is back-emf constant, and V_m is motor voltage. Rearrange (3) we get

$$V_m = \frac{R_m}{\eta_g \eta_m K_g K_t} \tau_m + K_g K_m \dot{\theta} \tag{4}$$

Equation (4) shows that the output voltage depends on the torque τ_m and the arm velocity $\dot{\theta}$.

5.1.2 State-Space Representation

Equations (1) and (2) show that the pendulum system is a non-linear system. However, non-linear systems are difficult to analyze, so we simplify the problem by linearizing the equations and get a collection of linear equations that approximates of the non-linear equations [1]. Consequently, we can define a state variable and utilize the state space representation, which is a standard form to describe multi-in multi-out dynamic system.

The state variable is defined as

$$\mathbf{x} = \begin{pmatrix} \theta \\ \alpha \\ \dot{\theta} \\ \dot{\alpha} \end{pmatrix} \tag{5}$$

where θ is the horizontal arm angle, $\dot{\theta}$ is the horizontal arm velocity, α is the pendulum angle, and $\dot{\alpha}$ is the pendulum velocity. Note that the encoders in the pendulum system only get the arm and pendulum angles. Consequently, only part of the state variable $\mathbf{x}$ is known and we will deal with it later.

The state-space representation of the inverted pendulum is

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{Ax} + \mathbf{B}u \\ \mathbf{y} &= \mathbf{Cx} + \mathbf{D}u \end{aligned} \tag{6}$$

where u is the voltage applied to the DC motor. In (6), $\mathbf{A}$ is a 4x4 square matrix, $\mathbf{B}$ is a 4x1 matrix, $\mathbf{C}$ is a 2x4 matrix, and $\mathbf{D}$ is a 2x1 matrix. The analytical solutions to these matrices are

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & a_{32} & a_{33} & a_{34} \\ 0 & a_{42} & a_{43} & a_{44} \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ b_3 \\ b_4 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \mathbf{D} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

where

$$a_{32} = \frac{m_p^2 l_p^2 r g}{J_{arm} m_p l_p^2 + J_{arm} J_p + m_p r^2 J_p}$$

$$a_{33} = -\frac{B_{arm}(J_p + m_p l_p^2)}{(J_{arm} m_p l_p^2 + J_{arm} J_p + m_p r^2 J_p) R_m}$$

$$a_{34} = \frac{m_p l_p r B_p}{J_{arm} m_p l_p^2 + J_{arm} J_p + m_p r^2 J_p}$$

$$a_{42} = \frac{m_p l_p g (J_{arm} + m_p r^2)}{J_{arm} m_p l_p^2 + J_{arm} J_p + m_p r^2 J_p}$$

$$a_{43} = -\frac{m_p l_p r B_{arm}}{(J_{arm} m_p l_p^2 + J_{arm} J_p + m_p r^2 J_p) R_m}$$

$$a_{44} = \frac{B_p (J_{arm} + m_p r^2)}{J_{arm} m_p l_p^2 + J_{arm} J_p + m_p r^2 J_p}$$

$$b_3 = \frac{(m_p l_p^2 + J_p)}{(J_{arm} m_p l_p^2 + J_{arm} J_p + m_p r^2 J_p) R_m}$$

$$b_4 = -\frac{m_p l_p r}{(J_{arm} m_p l_p^2 + J_{arm} J_p + m_p r^2 J_p) R_m}$$

Given the numerical solution to each variable in Appendix A, we can calculate the numerical solutions to the matrices. The results are

$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 53.1012 & -0.6586 & 0.6575 \\ 0 & 98.3814 & -0.6575 & 1.2182 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 274.4012 \\ 273.9627 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$\mathbf{D} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The advantage of the state space representation of the system is that it is easier to describe a MIMO system. And for a state-space system, the controller design can be separated into two steps. In the first step we assume that we can access all elements in the state variable **x**, i.e. we have a full state feedback. Based on this assumption we can design the full-state feedback controller. The second step deals with the fact that we are missing part of the elements in the state variable, i.e. $\dot{\theta}$ and $\dot{\alpha}$. To address this problem we design an observer to estimate the current state variable **x** based on the output from the system (i.e. θ and α over time).

5.1.3 Continuous to Discrete Domain Conversion

Equation (6) gives the continuous model of the pendulum system. However, when the controller is implemented on computers, it becomes a discrete (i.e. digital) controller. The system matrices need to be re-calculated when the controller is converted from continuous domain to discrete domain [2]. This section discusses how to do the conversion.

Consider the state space model (6). At $t = t_0$, given initial condition $\mathbf{x} = \mathbf{x}(t_0)$, the solution to the ordinary differential equation $\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}u$ is

$$\mathbf{x}(t) = e^{\mathbf{A}(t-t_0)}\mathbf{x}(t_0) + \int_{t_0}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}u(\tau)d\tau \tag{7}$$

In digital domain, the system would normally be sampled at a constant rate F , where F is the inverse of the sampling period T_s. Let $t_0 = nT_s$ and $t = (n+1)T_s$, then (1.7) becomes

$$\mathbf{x}((n+1)T_s) = e^{\mathbf{A}T_s}\mathbf{x}(nT_s) + \int_{nT_s}^{(n+1)T_s} e^{\mathbf{A}((n+1)T_s-\tau)}\mathbf{B}u(\tau)d\tau \tag{8}$$

If we use a zero-order hold (ZOH) sampler, then $u(\tau) = u(t_0) = u(nT_s)$. To facilitate the solution for a ZOH with no delay, let

$$\eta = (n+1)T_s - \tau \tag{9}$$

then we have

$$\mathbf{x}((n+1)T_s) = e^{\mathbf{A}T_s}\mathbf{x}(nT_s) + \int_0^{T_s} e^{\mathbf{A}\eta}d\eta\mathbf{B}u(nT_s) \tag{10}$$

In (10), if we define

$$\begin{aligned}\mathbf{F} &= e^{\mathbf{A}T_s}\\ \mathbf{G} &= \int_0^{T_s} e^{\mathbf{A}\eta}d\eta\mathbf{B}\\ \mathbf{H} &= \mathbf{C}\\ \mathbf{J} &= \mathbf{D}\end{aligned} \tag{11}$$

then we get

$$\begin{aligned}\mathbf{x}(n+1) &= \mathbf{F}\mathbf{x}(n) + \mathbf{G}u(n)\\ \mathbf{y}(n) &= \mathbf{H}\mathbf{x}(n) + \mathbf{J}u(n)\end{aligned} \tag{12}$$

Compare (11) with (6), we can see that (11) is the standard difference equations for a discrete system.

Using an infinite exponential series expansion, matrix $\mathbf{G}$ can be represented as

$$\mathbf{G} = \sum_{k=0}^{\infty} \frac{\mathbf{A}^k T_s^{k+1}}{(k+1)!}\mathbf{B} \tag{13}$$

Matlab can be used to convert a continuous system to a discrete system using the function *c2d*() as shown in Figure 2. The syntax of *c2d*() is sysd = c2d(sys, Ts, 'method'), where *sys* is the continuous system to be discretized, *Ts* is the sampling period, and *'method'* is the discretization method, which is chosen to be the zero-order hold (zoh) in the project.

```
% A,B,C,D are constant matrices for pendulum system
% Creates a system
pend=ss(A,B,C,D);

% then discrete the continuous system
pend_d=c2d(pend,Ts,'zoh');
```
Figure 2 Sample Code to Call c2d()

5.2 Balance Controller Design

Once the physical system has been represented, we must determine the amount of feedback gain to apply to the system. This section describes how to design the controller to balance the pendulum.

5.2.1 Full-State Feedback Controller Design

To determine the feedback gain, we first assume a full-state feedback, i.e. all the elements in the state variable **x** are known. Based on this assumption, the block diagram is shown in Figure 3.

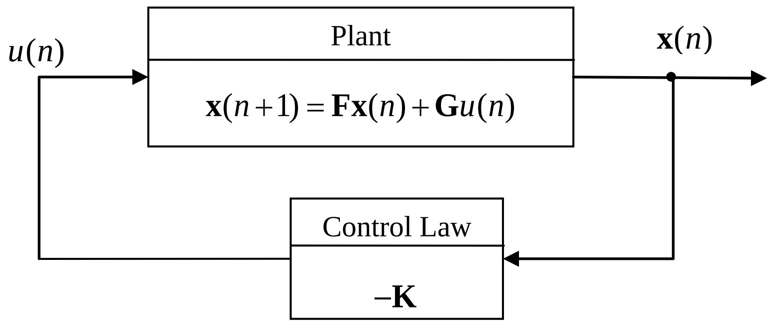


Figure 3 Block Diagram of the Full-State Feedback Control System

Here we use the linear quadratic regulator (LQR) algorithm to design the state feedback controller. For a continuous system $\dot{\mathbf{x}}=\mathbf{A}\mathbf{x}+\mathbf{B}u$, LQR theory states that the full-state feedback control system satisfies the following criteria [3]

(1) The closed-loop system is asymptotically stable

(2) The cost function, defined as $J=\int_0^\infty(\mathbf{x}^T\mathbf{Q}\mathbf{x}+u^T Ru)dt$, is minimized with feedback control law

$\mathbf{u}=-\mathbf{K}\mathbf{x}$

where $\mathbf{Q}$ is a nonnegative-definite matrix that penalizes the state variable $\mathbf{x}$ in the cost function , and R is a positive-definite matrix that penalizes the control input u in the cost function. Note that it follows that the LQR-based control design requires the availability of all the elements in the state variable $\mathbf{x}$. The feedback control gain $\mathbf{K}$ given by

$$\mathbf{K}=R^{-1}\mathbf{B}^T\mathbf{P} \tag{14}$$

where $\mathbf{P}$ is found by solving the continuous time algebraic Riccait equation

$$\mathbf{A}^T\mathbf{P}+\mathbf{P}\mathbf{A}-\mathbf{P}\mathbf{B}R^{-1}\mathbf{B}^T\mathbf{P}+\mathbf{Q}=\mathbf{0} \tag{15}$$

Similarly, for a discrete linear system described by $\mathbf{x}(n+1) = \mathbf{A}\mathbf{x}(n) + \mathbf{B}u(n)$, the cost function J is

$$J = \sum_{k=0}^{\infty} (\mathbf{x}(k)^T \mathbf{Q}\mathbf{x}(k) + u(k)^T Ru(k)) \tag{16}$$

and the feedback gain $\mathbf{K}$ can be calculated as [2]

$$\mathbf{K} = (R + \mathbf{B}^T\mathbf{P}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{P}\mathbf{A} \tag{17}$$

where $\mathbf{P}$ is the unique positive definite solution to the discrete time algebraic Riccati equation

$$\mathbf{P} = \mathbf{Q} + \mathbf{A}^T(\mathbf{P} - \mathbf{P}\mathbf{B}(R + \mathbf{B}^T\mathbf{P}\mathbf{B})^{-1}\mathbf{B}^T\mathbf{P})\mathbf{A} \tag{18}$$

We can use the Matlab function *dlqr*() to calculate $\mathbf{K}$. The syntax is [K,S,e] = dlqr(A,B,Q,R), where A and B is the matrix of discrete-time state-space system model, Q and R are the parameters defined in the cost function. Sample code is shown in Figure 4. Note that **F** and **G** are the same variables used in (11).

```
%% Design digital LQR controller
x = 0.2;    %weighting factor for the pendulum position
y = 0.4;    %weighting factor for the pendulum angle
Q = [x 0 0 0;
     0 y 0 0;
     0 0 0 0;
     0 0 0 0];
R = 5;
K_controlgain = dlqr(F,G,Q,R)
```

Figure 4 Sample Code to Call dlqr()

5.2.2 Observer Design

As indicated above, we do not have direct access to the complete state variable. Consequently, we must estimate the missing values, which can be done using an observer [3]. The general form of the state variable feedback compensator is

$$\dot{\hat{\mathbf{x}}} = \mathbf{A}\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}}) \tag{19}$$

where $\hat{\mathbf{x}}$ denotes the estimate of the state variable **x**, and $\mathbf{L}$ is a matrix representing a collection of coefficients to be determined.

If we define the estimation error, **e**, as

$$\mathbf{e} = \mathbf{x} - \hat{\mathbf{x}} \tag{20}$$

and (6)-(19), we get

$$\begin{aligned} \dot{\mathbf{x}} - \dot{\hat{\mathbf{x}}} &= \mathbf{Ax} + \mathbf{B}u - (\mathbf{A}\hat{\mathbf{x}} + \mathbf{B}u + \mathbf{L}(\mathbf{y} - \mathbf{C}\hat{\mathbf{x}})) \\ &= \mathbf{A}(\mathbf{x} - \mathbf{A}\hat{\mathbf{x}}) - \mathbf{L}(\mathbf{Cx} - \mathbf{C}\hat{\mathbf{x}}) \\ &= (\mathbf{A} - \mathbf{LC})(\mathbf{x} - \hat{\mathbf{x}}) \end{aligned} \tag{21}$$

Consequently

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{LC})\mathbf{e} \tag{22}$$

Control theory indicates that if all the poles (i.e. roots) of system (22) locate inside the unit circle, the system is stable. Therefore, given a vector $\mathbf{P}$ of desired, self-conjugate, closed-loop pole locations, the matrix $\mathbf{L}$ can be determined by calling Matlab function *place*(). The syntax is K = place(A, B, p), where A and B are the matrices of discrete-time state-space system model, and p is the vector for pole locations.

In the discrete domain, the observer design is similar. The equation for the observer is

$$\hat{\mathbf{x}}(n+1) = \mathbf{F}\hat{\mathbf{x}}(n) + \mathbf{G}u(n) + \mathbf{L}(\mathbf{y}(n) - \mathbf{H}\hat{\mathbf{x}}(n)) \tag{23}$$

and the estimation error can be defined as

$$\mathbf{e}(n) = \mathbf{x}(n) - \hat{\mathbf{x}}(n) \tag{24}$$

Then (11)-(23) we get

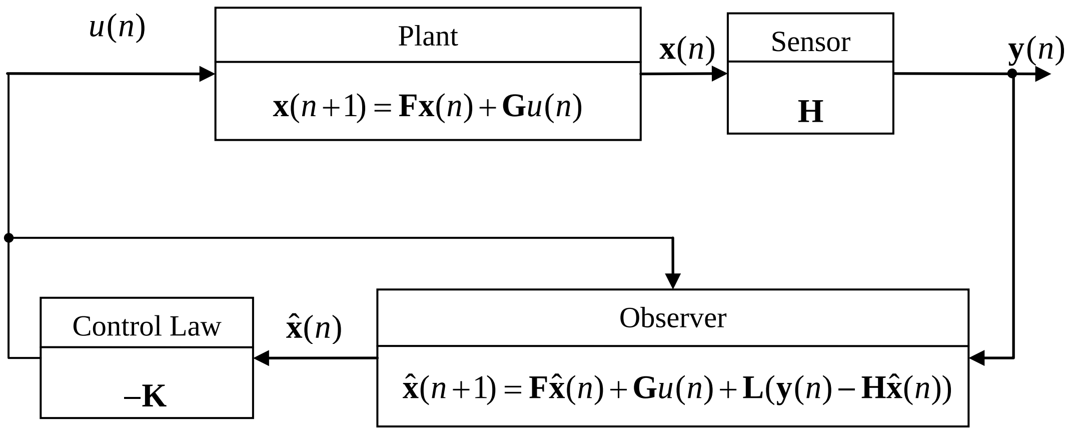
$$\begin{aligned} \mathbf{e}(n+1) &= \hat{\mathbf{x}}(n+1) - \mathbf{x}(n+1) \\ &= \mathbf{Fx}(n) + \mathbf{G}u(n) - (\mathbf{F}\hat{\mathbf{x}}(n) + \mathbf{G}u(n) + \mathbf{L}(\mathbf{y}(n) - \mathbf{C}\hat{\mathbf{x}}(n))) \\ &= (\mathbf{F} - \mathbf{LH})(\mathbf{x}(n) - \hat{\mathbf{x}}(n)) \\ &= (\mathbf{F} - \mathbf{LH})\mathbf{e}(n) \end{aligned} \tag{25}$$

We can also call place() to determine the matrix **L** as shown in Figure 5, where F and H represents the matrices in (1.11), and F' represents the transpose of F.

```
    % pole locations
    P = [-0.3; -0.3; -0.9; -0.9];
    % pole placement
    L = place (F',H',P)';
```

Figure 5 Sample Code to Call place()

Therefore, the block diagram of the control system based on (23) is shown in Figure 6.



$u(n)$
Plant
$\mathbf{x}(n+1)=\mathbf{F}\mathbf{x}(n)+\mathbf{G}u(n)$
$\mathbf{x}(n)$
Sensor
$\mathbf{H}$
$\mathbf{y}(n)$
Control Law
$-\mathbf{K}$
$\hat{\mathbf{x}}(n)$
Observer
$\hat{\mathbf{x}}(n+1)=\mathbf{F}\hat{\mathbf{x}}(n)+\mathbf{G}u(n)+\mathbf{L}(\mathbf{y}(n)-\mathbf{H}\hat{\mathbf{x}}(n))$

Figure 6 Block Diagram of the Control System without Reference Signal

5.2.3 Controller with Reference Signal Input

A limitation of the above controller is that it does not accept a "reference input". In the general case, a "reference input" is a desired output that the system will converge to. Recall that for the rotary inverted pendulum, the output **y** is $\begin{pmatrix}\theta \\ \alpha\end{pmatrix}$, which has 2 elements. In order to keep the pendulum balanced, the pendulum angle α should be 0 all the time, so that the only variable that we can control is the horizontal arm angle θ. In this case, we only set the reference input to a scalar. This section discusses how to design the controller in the presence of a reference input.

Let us first repeat the plant (12) and controller equations (23), where the plant refers to the system being controlled, i.e. the pendulum

$$\left.\begin{aligned}\mathbf{x}(n+1)&=\mathbf{F}\mathbf{x}(n)+\mathbf{G}u(n)\\ \mathbf{y}(n)&=\mathbf{H}\mathbf{x}(n)\end{aligned}\right\}\textit{plant} \tag{26}$$

$$\left.\begin{aligned}\hat{\mathbf{x}}(n+1)&=(\mathbf{F}-\mathbf{G}\mathbf{K}-\mathbf{L}\mathbf{H})\hat{\mathbf{x}}(n)+\mathbf{L}\mathbf{y}(n)\\ u(n)&=-\mathbf{K}\hat{\mathbf{x}}(n)\end{aligned}\right\}\textit{controller} \tag{27}$$

Therefore, if we want to control the horizontal arm angle, the controller receives a reference input $r(n)$ and (27) becomes

$$\left.\begin{aligned}\hat{\mathbf{x}}(n+1)&=(\mathbf{F}-\mathbf{G}\mathbf{K}-\mathbf{L}\mathbf{H})\hat{\mathbf{x}}(n)+\mathbf{L}\mathbf{y}(n)+\mathbf{M}r(n)\\ u(n)&=-\mathbf{K}\hat{\mathbf{x}}(n)+Nr(n)\end{aligned}\right\}\textit{controller} \tag{28}$$

where $\mathbf{M}$ (a matrix) and N (a vector) are to be determined. To determine **M** and *N*, the criterion is that $r(n)$ does not have any influence on the system, which means that the state error, $\mathbf{e}(n)=\mathbf{x}(n)-\hat{\mathbf{x}}(n)$, is independent of $r(n)$.

Since

$$\begin{aligned}\mathbf{x}(n+1)&=\mathbf{F}\mathbf{x}(n)+\mathbf{G}u(n)\\ &=\mathbf{F}\mathbf{x}(n)+\mathbf{G}(-\mathbf{K}\hat{\mathbf{x}}(n)+Nr(n))\\ &=\mathbf{F}\mathbf{x}(n)-\mathbf{G}\mathbf{K}\hat{\mathbf{x}}(n)+N\mathbf{G}r(n)\end{aligned} \tag{29}$$

and

$$\begin{aligned}\hat{\mathbf{x}}(n+1)&=(\mathbf{F}-\mathbf{G}\mathbf{K}-\mathbf{L}\mathbf{H})\hat{\mathbf{x}}(n)+\mathbf{L}\mathbf{y}(n)+\mathbf{M}r(n)\\ &=(\mathbf{F}-\mathbf{G}\mathbf{K}-\mathbf{L}\mathbf{H})\hat{\mathbf{x}}(n)+\mathbf{L}\mathbf{H}\mathbf{x}(n)+\mathbf{M}r(n)\end{aligned} \tag{30}$$

(29)- (30) we get

$$\mathbf{x}(n+1)-\hat{\mathbf{x}}(n+1)=(\mathbf{F}-\mathbf{L}\mathbf{H})(\mathbf{x}(n)-\hat{\mathbf{x}}(n))+(N\mathbf{G}-\mathbf{M})r(n) \tag{31}$$

Substituting $\mathbf{e}(n)=\hat{\mathbf{x}}(n)-\hat{\mathbf{x}}(n)$ we get

$$\mathbf{e}(n+1)=(\mathbf{F}-\mathbf{L}\mathbf{H})\mathbf{e}(n)+(N\mathbf{G}-\mathbf{M})r(n) \tag{32}$$

In (32), if $r(n)$ has no effect on $\mathbf{e}(n)$, it must be true that $N\mathbf{G}-\mathbf{M}=0$, therefore

$$\mathbf{M}=N\mathbf{G} \tag{33}$$

Since $\mathbf{G}$ is known, the only thing we need to determine is N, the reference gain. If we assume that $\mathbf{x}(n)=\hat{\mathbf{x}}(n)$ for a sufficient large *n*, which indicates that the estimated state converges to the real pendulum state after sufficient long time, then we get

$$\begin{aligned}\mathbf{x}(n+1) &= \mathbf{F}\mathbf{x}(n)+\mathbf{G}u(n)\\ &= \mathbf{F}\mathbf{x}(n)+\mathbf{G}(Nr(n)-\mathbf{K}\hat{\mathbf{x}}(n))\\ &= \mathbf{F}\mathbf{x}(n)+\mathbf{G}(Nr(n)-\mathbf{K}\mathbf{x}(n))\\ &= (\mathbf{F}-\mathbf{G}\mathbf{K})\mathbf{x}(n)+N\mathbf{G}r(n)\end{aligned} \tag{34}$$

Take the *z*-transform on both sides

$$z\mathbf{X}(z)=(\mathbf{F}-\mathbf{G}\mathbf{K})\mathbf{X}(z)+N\mathbf{G}R(z) \tag{35}$$

Solve $\mathbf{X}(z)$ and get

$$\mathbf{X}(z)=(z\mathbf{I}-\mathbf{F}+\mathbf{G}\mathbf{K})^{-1}N\mathbf{G}R(z) \tag{36}$$

Consequently, the *z*-transform of the output $\mathbf{y}(n)$ is

$$\begin{aligned}\mathbf{Y}(z) &= \mathbf{H}\mathbf{X}(x)\\ &= \mathbf{H}(z\mathbf{I}-\mathbf{F}+\mathbf{G}\mathbf{K})^{-1}N\mathbf{G}R(z)\end{aligned} \tag{37}$$

If we assume that the reference input is a step signal, which means that $r(n)=1$ for $n\geq 0$. For the step signal, the z-transform is $R(z)=z/(z-1)$, we can apply the final value theorem to (1.36) and get

$$\begin{aligned}\lim_{k\to\infty}\mathbf{y}(k) &= \lim_{z\to 1}(z-1)\mathbf{Y}(z)\\ &= \lim_{z\to 1}(z-1)\mathbf{H}(z\mathbf{I}-\mathbf{F}+\mathbf{G}\mathbf{K})^{-1}N\mathbf{G}z/(z-1)\\ &= N\mathbf{H}(\mathbf{I}-\mathbf{F}+\mathbf{G}\mathbf{K})^{-1}\mathbf{G}=\begin{pmatrix}1\\0\end{pmatrix}\end{aligned} \tag{38}$$

and we determine the numerical value of the reference gain *N* from (38) by calculating matrix **U**

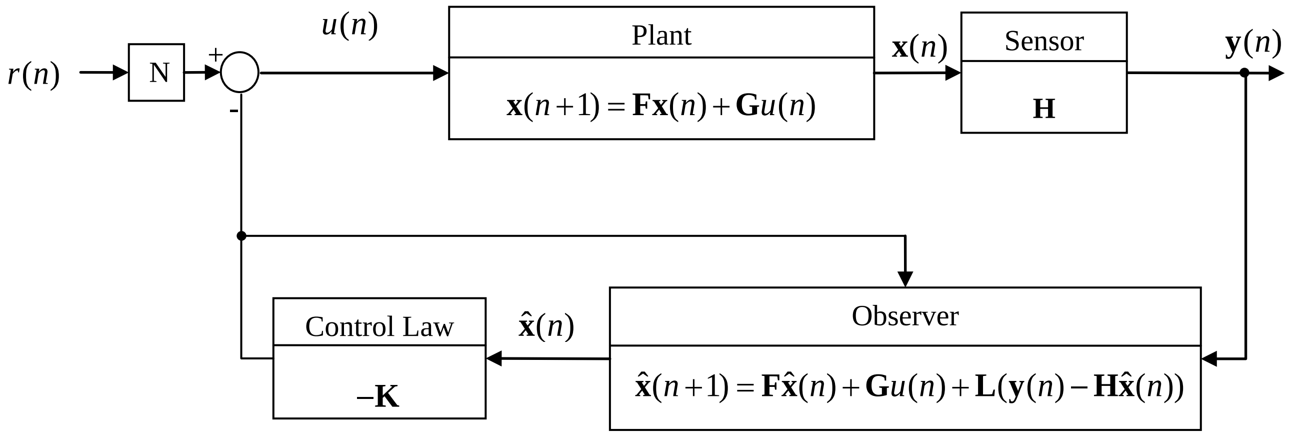
$$\mathbf{U}=\mathbf{H}(\mathbf{I}-\mathbf{F}+\mathbf{G}\mathbf{K})^{-1}\mathbf{G} \tag{39}$$

and

$$N=\frac{1}{\mathbf{U}(1)} \tag{40}$$

where **U**(1) represents the first row of matrix **U.**

Therefore, the resulting control system block diagram with a reference signal is designed in Figure 7.



$r(n)$
N
+
-
$u(n)$
Plant
$\mathbf{x}(n+1)=\mathbf{F}\mathbf{x}(n)+\mathbf{G}u(n)$
$\mathbf{x}(n)$
Sensor
H
$\mathbf{y}(n)$
Control Law
$-\mathbf{K}$
$\hat{\mathbf{x}}(n)$
Observer
$\hat{\mathbf{x}}(n+1)=\mathbf{F}\hat{\mathbf{x}}(n)+\mathbf{G}u(n)+\mathbf{L}(\mathbf{y}(n)-\mathbf{H}\hat{\mathbf{x}}(n))$

Figure 7 Block Diagram of the Control System with Reference Signal

The observer can be implemented based on its state-space representation, which is

$$\begin{aligned}\dot{\hat{\mathbf{x}}}&=\hat{\mathbf{A}}\hat{\mathbf{x}}+\hat{\mathbf{B}}\hat{\mathbf{u}}\\ \hat{\mathbf{y}}&=\hat{\mathbf{C}}\hat{\mathbf{x}}+\hat{\mathbf{D}}\hat{\mathbf{u}}\end{aligned} \tag{41}$$

where the state variable $\hat{\mathbf{x}}$ is the estimation of **x**.

Note that in Figure 7, the observer has two inputs: *u* and **y**, so define the input for the observer as $\hat{\mathbf{u}}=\begin{pmatrix}u\\ \mathbf{y}\end{pmatrix}$, and the other matrices are

$$\hat{\mathbf{A}} = (\mathbf{A}-\mathbf{LC}), \hat{\mathbf{B}} = (\mathbf{B} \quad \mathbf{L}), \hat{\mathbf{C}} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \hat{\mathbf{D}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

5.3 Simulation

Section 5.2 gives the theoretical solution to the controller. Based on the control block diagram in Figure 7, we can implement the control system in Simulink, and simulate the performance of the controller.

5.3.1 Simulink Simulation Model

The Simulink representation of the complete model is shown in Figure 8, in which each rectangle is a Simulink block or subsystem. For example, the block “rotary pendulum” fully characterizes the pendulum system. The State Space Observer subsystem characterizes the Observer. The detail of the State Space Observer subsystem is shown in Figure 9.

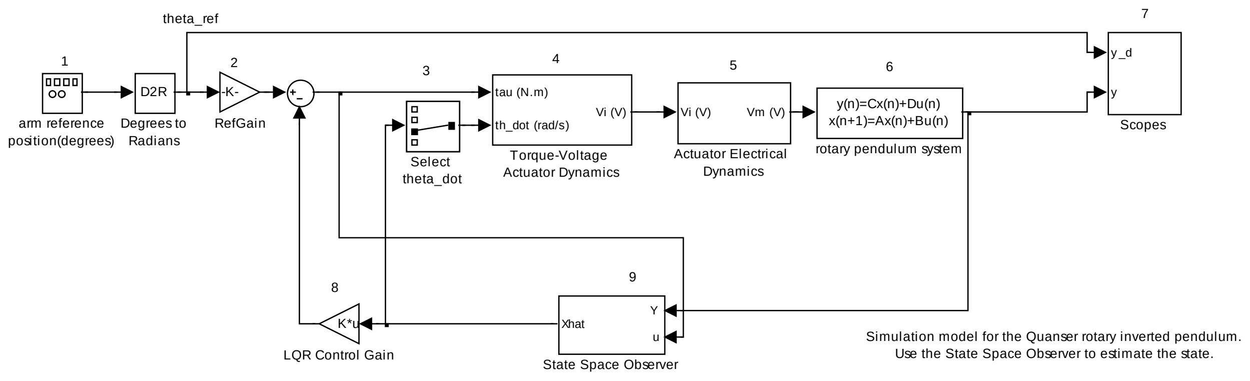


Figure 8 Simulink Model with Motor Dynamics

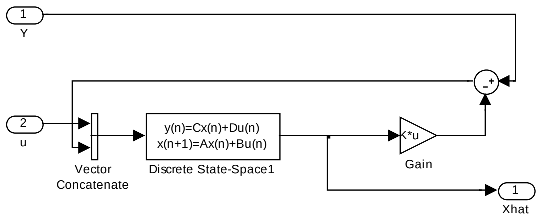


Figure 9 State-Space Observer

5.3.2 Parameters

Parameters that can be adjusted include the sampling period, the **Q** and *R* matrices, and the poles' angles. Configure the system with the following parameters:

- sampling period T_s=0.001s
- LQR algorithm matrices Q=[0.2 0 0 0;0 0.4 0 0; 0 0 0 0;0 0 0 0], R = 5
- Poles are at [-0.3; -0.3; -0.9; -0.9].

Once we have the Simulink model, the controller can be can simulated.

The first experiment tests whether the controller can balance the pendulum. Set the reference signal to 0, which means that we keep the horizontal arm at a fixed angle of 0 degree. Assume that the initial state of the pendulum is

$$\mathbf{x}_0 = \begin{pmatrix} 0.1 \\ 0.1 \\ 0.5 \\ 0.5 \end{pmatrix}$$

which means that initially the vertical pendulum and the horizontal arm are at 0.1 rad (5.7 degrees), and their velocities are 0.5 rad/s. Apparently, the pendulum deviates from the up-right position and is rotating, so it is not balanced. The controller works at the sampling rate of 1KHz. Figure 10 shows the arm and pendulum angles changing over time. From the figure, we can see that at t=0, the values of α

and θ are 5.7. Within 3 seconds, both α and θ converges to 0, which means that the pendulum is balanced.

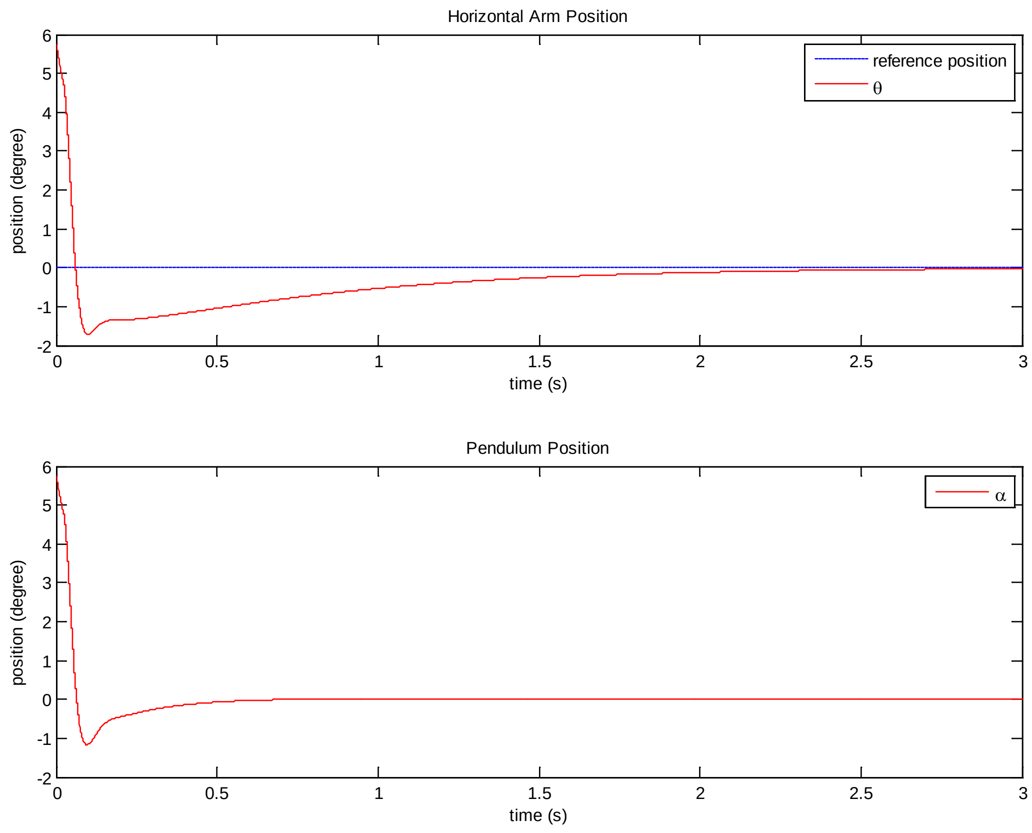


Figure 10 Simulation Result when X0=[0.1;0.1;0.5;0.5]

The second experiment tests whether the horizontal arm angle can follow the reference signal. Set the reference input to a square wave with an amplitude of 0.1 rad (5.73 degrees), and a frequency of 0.1 Hz. Figure 11 shows the results of the pendulum angle and arm angle. We can see that the arm angle tracks the reference input and keeps the pendulum angle within 1.5 degrees, which indicates the pendulum is balanced.

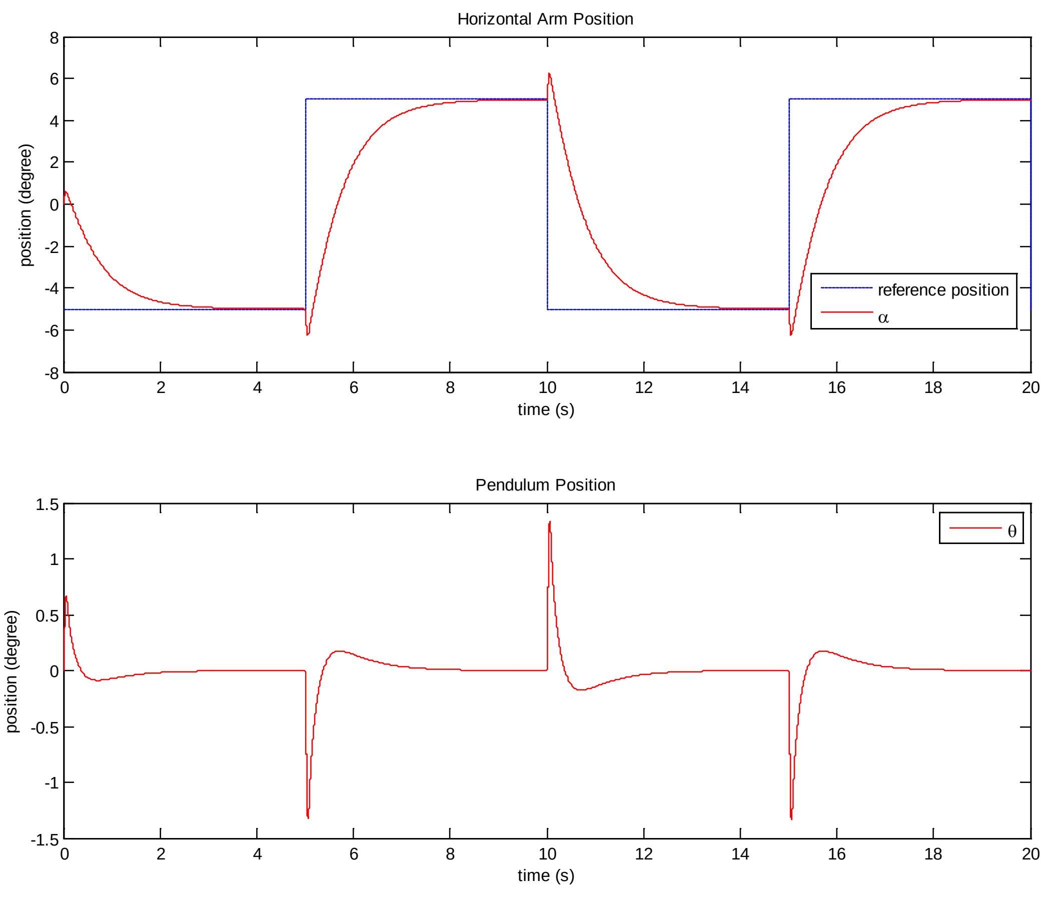


Figure 11 Pendulum System Simulation Result

5.3.3 Networked Control System Model

To simulate a networked control system, a model was created (Figure 12) using the TrueTime toolbox [4]. The TrueTime Network block (number 11 in Figure 12) can be configured to be one of several networks (e.g. Ethernet, CAN), and its bit rate or loss rate.

In Figure 12, subsystems 1-6 are the same as those of the control system in Figure 8 (i.e. the local control system). But it is divided into two isolated systems: the controller system and the pendulum system. The pendulum system (blocks 1-4) communicates with the controller (blocks 5,6,8) using the facilities of the TrueTime send/receive blocks (blocks 7-10).

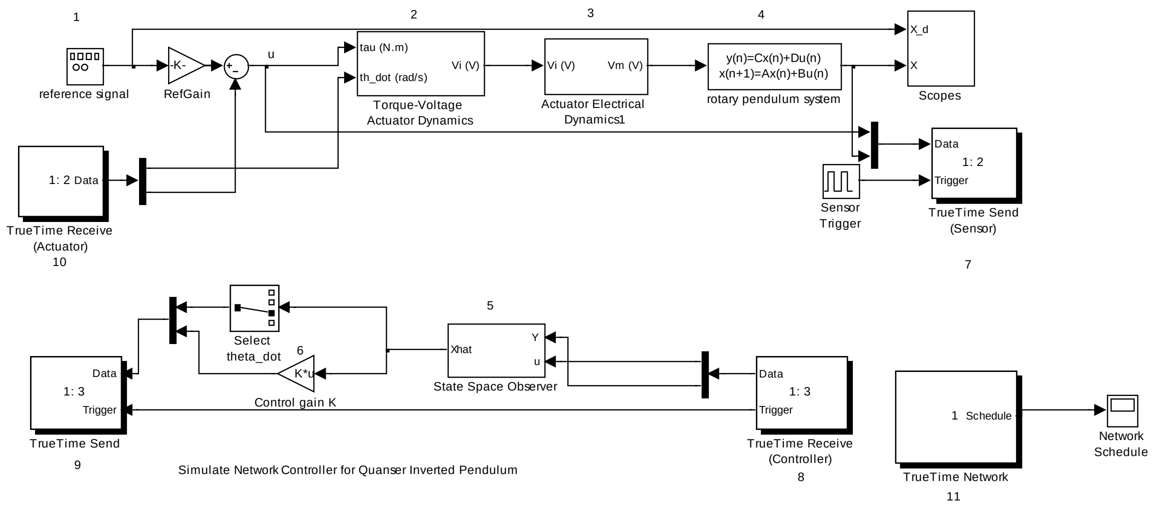


Figure 12 NCS Simulink Model

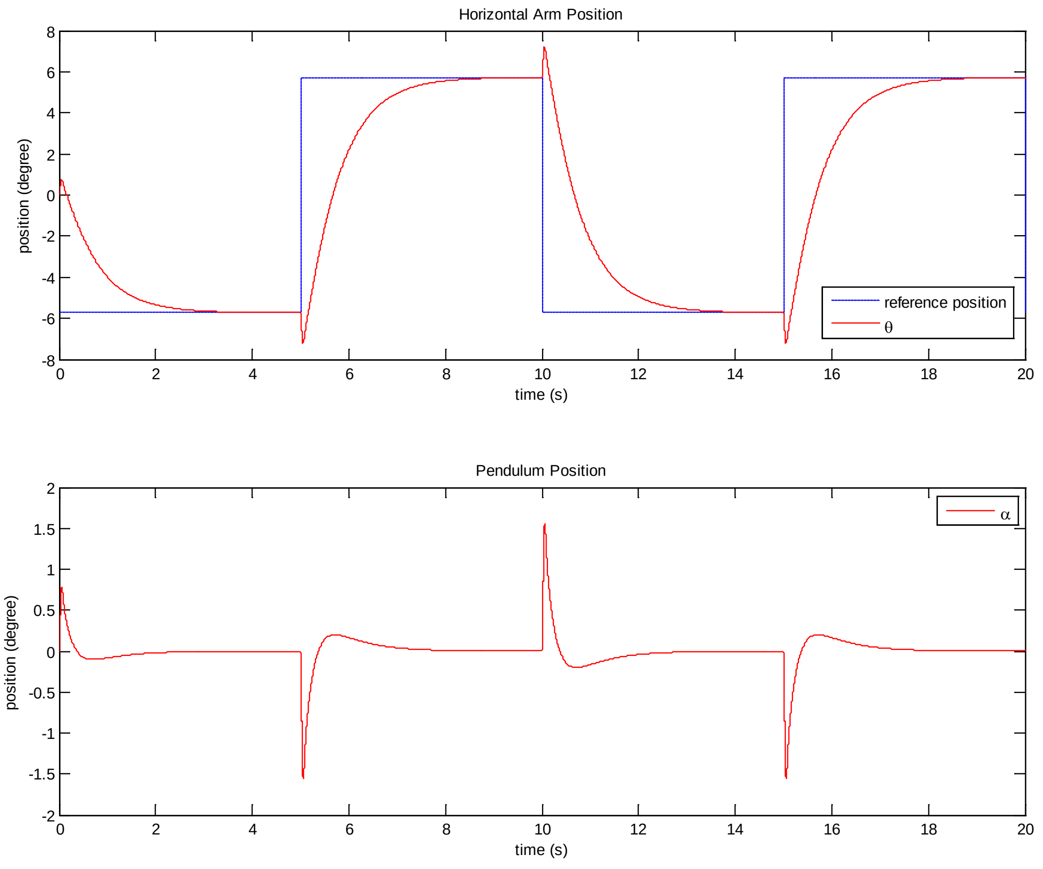


Figure 13 NCS Simulation Result

The pendulum system sends get the angle information and to the controller, where the data rate is controlled by the sensor trigger. The pendulum receives the control signal and writes it to the actuator. The controller system receives the angle information, calculates the control signal, and sends it back to the actuator through controller sender (block 9).

Simulate the network control system using the same parameters listed in 5.3.2, and the simulation result is shown in Figure 13. Compare Figure 13 and Figure 11, we can see that the maximum pendulum angle in Figure 13 is 1.5 degrees, which is larger than that in Figure 11. This may be because of the delay effect in the network.

5.4 Simulink Executable Model

Using Quanser QUARC library, we can compile a Simulink model into a QNX executable image that can be used to control the actual system. This executable model is implemented in Figure 14. The structure of the executable model was developed based on the Simulink model shown in Figure 8.

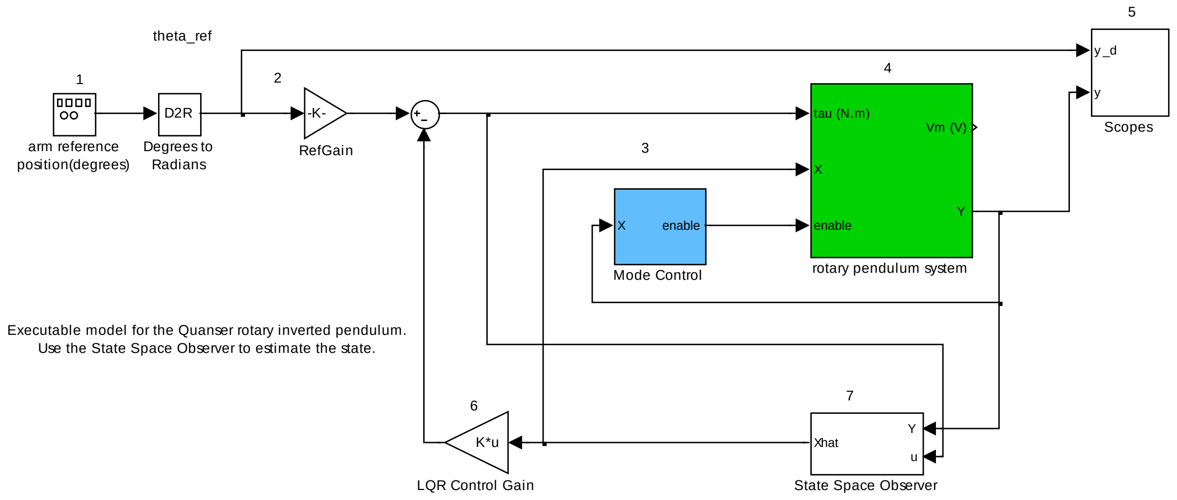


Figure 14 Executable Simulink Model

Compare Figure 14 with Figure 8, we see that a *Mode Control* (MC) subsystem is present in Figure 14. The MC receives the output of the pendulum and output an enable signal, which controls when the balance controller begins to execute. The MC subsystem is necessary because that the encoders only give the relative angle, not the absolution angle. This is the limitation of the encoders. When powered on, the encoders read 0. If the pendulum is at arbitrary position, we cannot tell the exact angle. However, there is one deterministic position, i.e. when the pendulum is static, it must be in the straight down position, which is 180 degrees. Therefore, the MC subsystem utilized this information. When the model begins to execute, the MC subsystem assumes that the pendulum is static, and wait for the user to swing it up. The controller can only work when the pendulum is close to the up-right position. The threshold is defined as 2.5 degrees. If the pendulum angle is larger than the threshold, the MC subsystem outputs 0 to disable the controller. Otherwise, the MC subsystem outputs 1 to enable the controller. The MC subsystem is designed as in Figure 15.

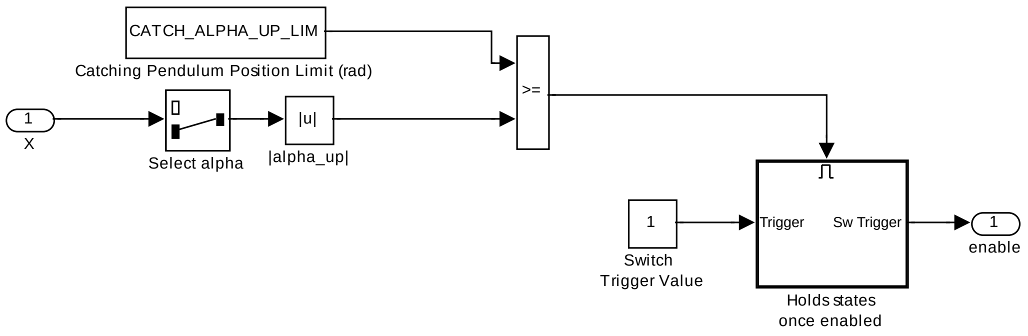


Figure 15 Mode Control Subsystem

The pendulum system sub-system is the one that directly controls the Q4 board and the power module. Figure 17 shows the details of the subsystem. The *HIL initialize* block will initialize the Q4 board when the model begins to execute. The *HIL Read Encoder* block reads encoder values. The *Torque-Voltage Actuator Dynamics* and *Actuator Electrical Dynamics* subsystems convert the input torque to the desired voltage, which will be output by the *HIL Write Analog* block to the power amplifier.

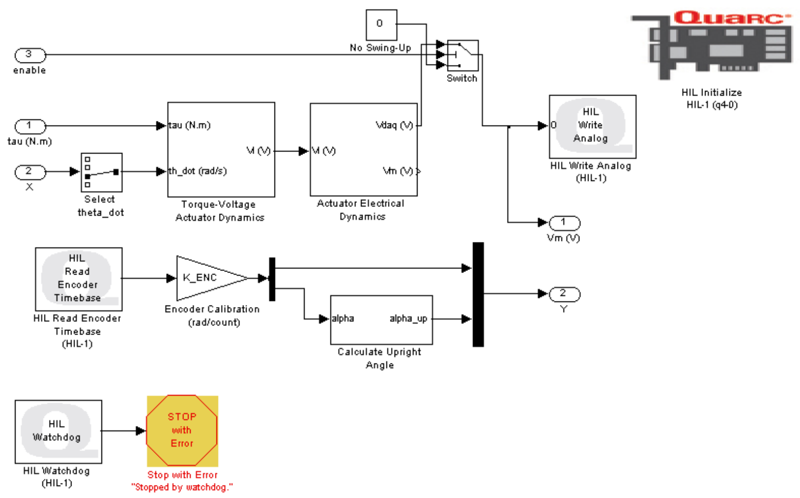


Figure 16 Pendulum System Subsystem

The Calculate Upright Angle subsystem (Figure 17) is used to convert the pendulum angles from its encoder values. When the pendulum is initially at its straight-down position, the encoder reads 0, which is mapped to 180 degrees. When the pendulum is swung up, the encoder reads either 2048 or -2048, which is mapped to 0 degree. This subsystem apply the modulo operation to the input angle, and subtract the offset angle to get the absolution angle.

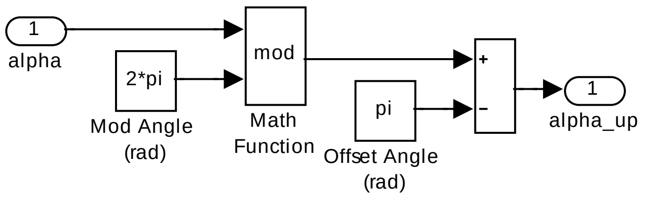


Figure 17 Calculate Upright Angle Subsystem

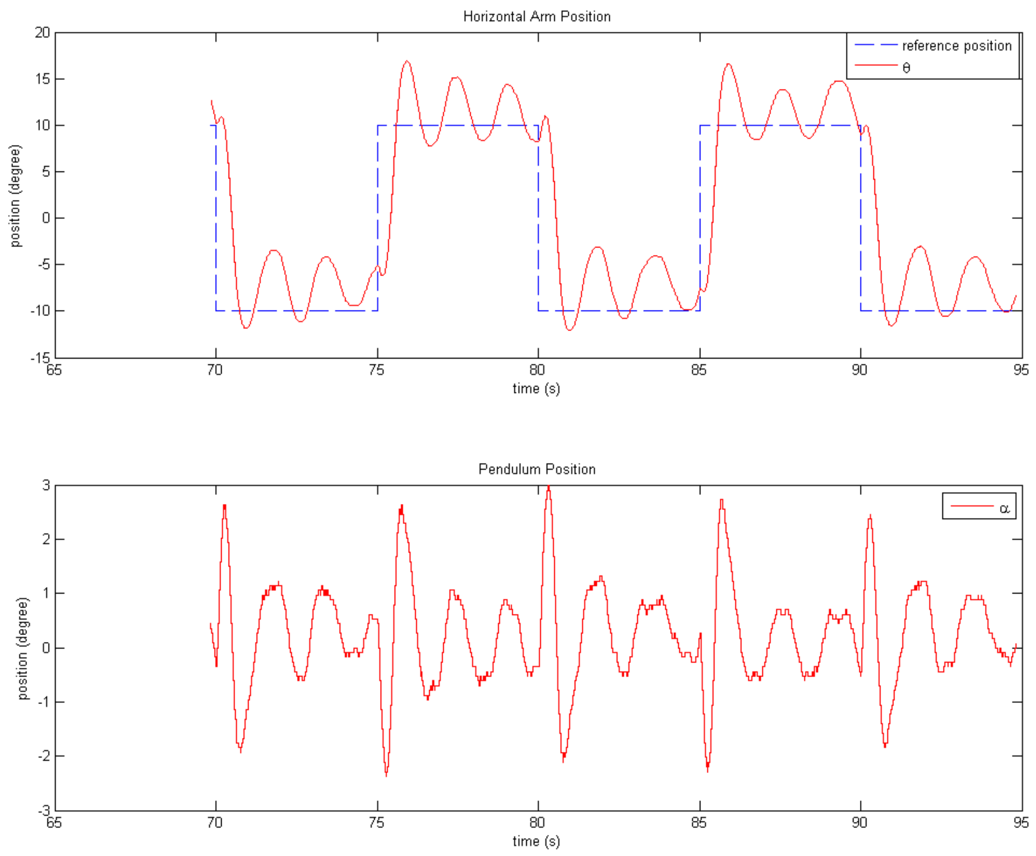


Figure 18 Running Result

Compile and run the model, we can see that the controller can balance the pendulum. Figure 18 records the arm and pendulum angles during an experiment. The horizontal arm is tracking the input reference signal, which is a square wave with amplitude of 10 and frequency of 0.1 Hz.

5.5 Summary

This chapter introduces how the pendulum controller is designed. The general process is as follows. First, analyze the mathematical representation of the physical system. Second, design the control system block diagram based on control theory. Third, create the Simulink model based on the control block diagram, and tune the parameters to make sure that the controller is working properly. Forth, implement the executable controller by utilizing the Quanser library. In the end, compile and run it on hardware.

References

[1] Quanser Inc., "Rotary Experiment #08: Self Erecting Inverted Pendulum Control," .

[2] G. Franklin and D. Powell, *Digital Control of Dynamic Systems.* Addison-Wesley publishing company, 1980.

[3] R. Dorf and R. Bishop, *Modern Control Systems.* Prentice Hall, 2008.

[4] Department of Automatic Control, Lund University. TrueTime toolbox. [online]. *2.0 beta 6* Available: http://www.control.lth.se/user/truetime/.

6 C++ Project

Though Quanser QUARC library and Simulink provides a fast way to construct a controller, the drawback is that the sampling rate cannot be varied in the Simulink model, because it is a fixed environment parameter. This restricts the further research on the time scales control theory, which requires the sampling rate to be altered during runtime. The time scales can be generated by hardware, i.e. utilizing an external signal generator. However, it is still difficult to directly control the signal generator in Simulink. Therefore, I created a software solution by implementing the controller in C++.

6.1 C++ Version of the Balance Controller

The C++ version of the pendulum controller is implemented by mimicking the Simulink model. The C++ code implements all the functions that are in the executable Simulink model. The executable Simulink model is redrawn in Figure 1, in which subsystems that pertain to the same functional group are circled in red circle.

The reference signal group generates a proper reference signal, and applies the correct gain to eliminate the steady-state error. The controller group determines the status of the pendulum, uses the observer to estimate the current state variable, and applies the LQR feedback control law. The Q4 board group directly accesses the Q4 board to read/write data.

The C++ program can be modeled using SysML. The Block Definition Diagram (BDD) describes the general structure of the system. The BDD of the pendulum system is shown in Figure 2. The PendulumSystem class is the top-level class that is composed of three main classes: Reference Signal, Controller, and Q4. The PendulumSystem class stores the state variable and other variables that are exchanged between different classes and are frequently updated. The reason is that the difference equation $\mathbf{x}(n+1)=\mathbf{F}\mathbf{x}(n)+\mathbf{G}u(n)$ indicates that the next pendulum state variable $\mathbf{x}(n+1)$ is calculated based on the current input $u(n)$, and the current state variable $\mathbf{x}(n)$, which should be saved. Therefore, I designed the PendulumSystem class to be the top class that keeps track of everything.

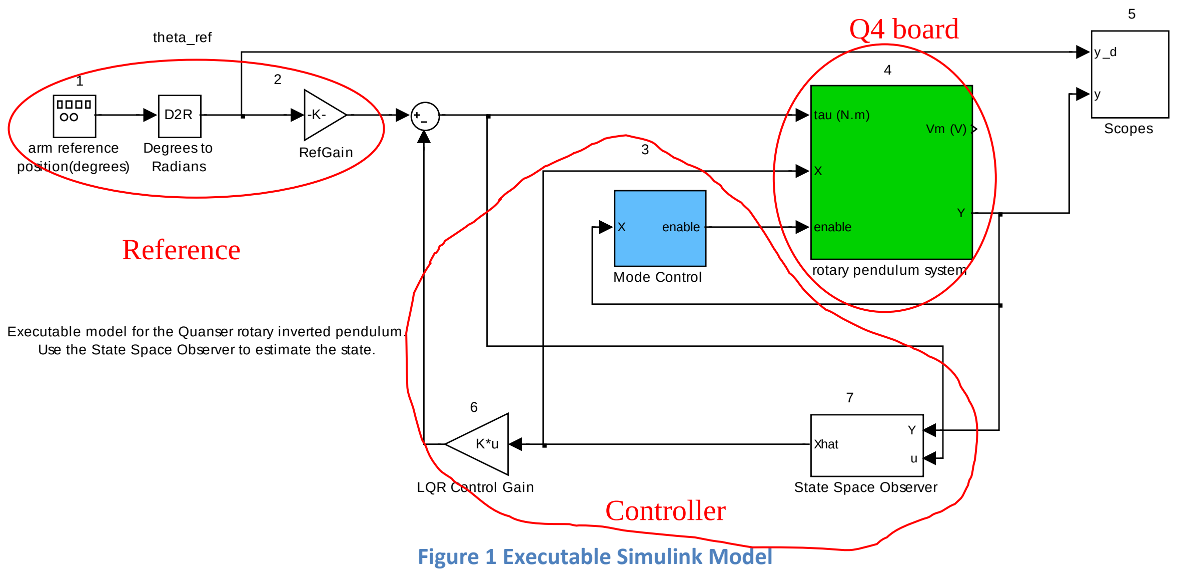


Figure 1 Executable Simulink Model

The internal structure is best described using the Internal Block Diagram (IBD). The advantage of SysML is that we can show the interfaces and data flow between each subsystem. As shown in Figure 3, the Reference Signal class outputs angle data (i.e. the horizontal arm position) to the Controller block. The Q4 block outputs encoder data (in the form of a Matrix) to the Controller block. Based on these information, the Controller calculates the voltage to be applied to the DC motor, and outputs it to the Q4 block.

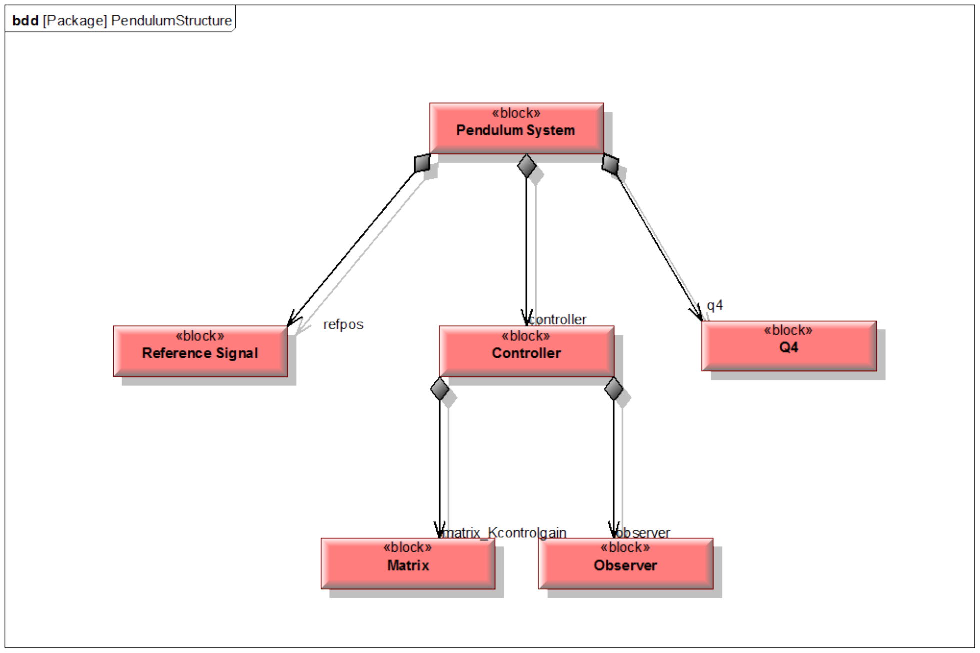


Figure 2 BDD of pendulum system

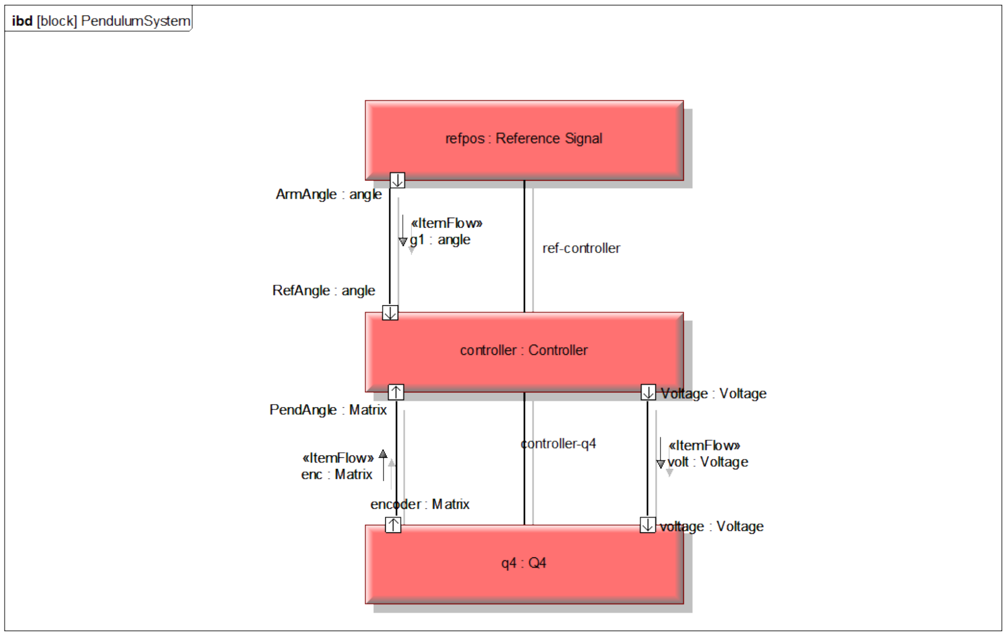


Figure 3 IBD of PendulumSystem block

6.2 Varying Sampling Rate

Varying sampling rate is achieved by using a QNX software timer. The QNX provides software timers whose expiration time could be set during runtime. At each timer expiration, the QNX OS sends a signal to the C++ grogram, which is waiting for that signal. Upon receiving the signal, the main() function will call the update() function in the PendulumSystem class, which reads encoder data, calculates the control signal (voltage), and outputs the voltage to the Q4 class.

The PendulumSystem::update() function accepts an input parameter of Boolean type, which indicates whether the controller runs at a fixed rate or a variable rate. The return value indicates what the next sampling period is (Figure 5), so the software time can be set. In this way, the timer expiration time it set at each step so that the sampling rate is varying. This gives us the flexibility to generate time scales.

The implementation of the PendulumSystem::update() function is shown in Figure 4. If the fixed step length flag is to false, the function calculates the Euclidian norm of the current estimated state variable $\hat{\mathbf{x}}$, then determine the next sampling period based on that norm.

```
    //if using time scales control, determine next sampling
period
    if(!isFixedRate)
    {
            //calculate norm of the vector
            double dp = matrix_Xhat_.norm();

            int index = 0;
            if(dp > 2)
            {
                    nextTs = constants::TS5MS;
            }
            else
            {
                    nextTs = constants::TS20MS;
                    index = 1;
            }
            controller.loadParameters(index);
            refpos.loadParameters(index);
    }
```

Figure 4 int PendulumSystem::update(bool isFixedRate)

```
    int Ts = mysys.update(true);

    //set timer
    //expire time = Ts
    itime.it_value.tv_sec = 0;
    itime.it_value.tv_nsec = Ts;
    //reload time = Ts
    itime.it_interval.tv_sec = 0;
    itime.it_interval.tv_nsec = Ts;
    timer_settime(timer_id, 0, &itime, NULL);
```

Figure 5 Calling PendulumSystem::update()

7 Creating your own controller

When you need to create your own controller, you can modify the existing Simulink and QNX C++ project.

7.1 Step 1

The first step is to create a simulation model in Simulink. This requires you to modify file sim_statespace_obeserver.mdl, as shown in Figure 1.

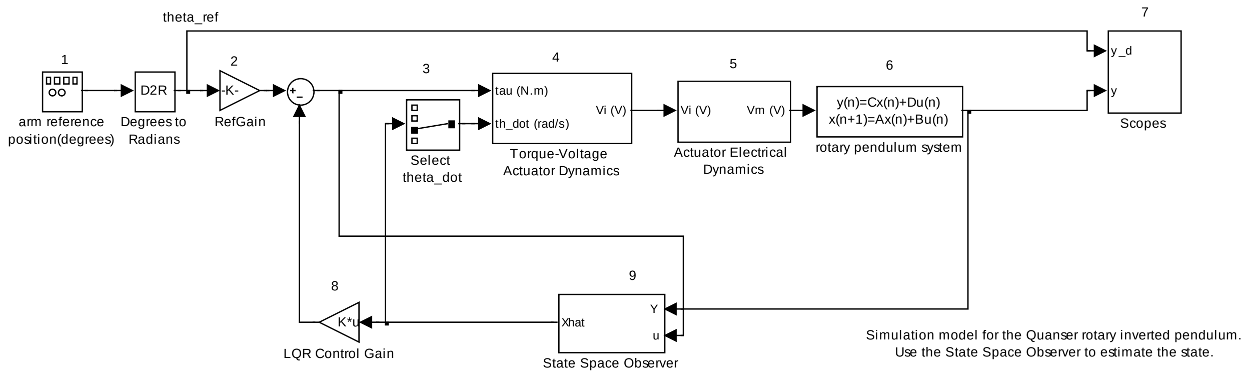


Figure 1 sim_statespace_obeserver.mdl

You should keep block 1, 4, 5 and 6. Block 1 generates and outputs the reference angle. Block 4 and 5 are for motor dynamics, which converts the torque to voltage. Block 6 is the physical model of the pendulum system, which accepts voltage and outputs the arm and pendulum angles.

Then create your own controller by adding the controller blocks into the diagram.

7.2 Step 2

After you get successful simulation results, you can modify file exe_statespace_obeserver.mdl.

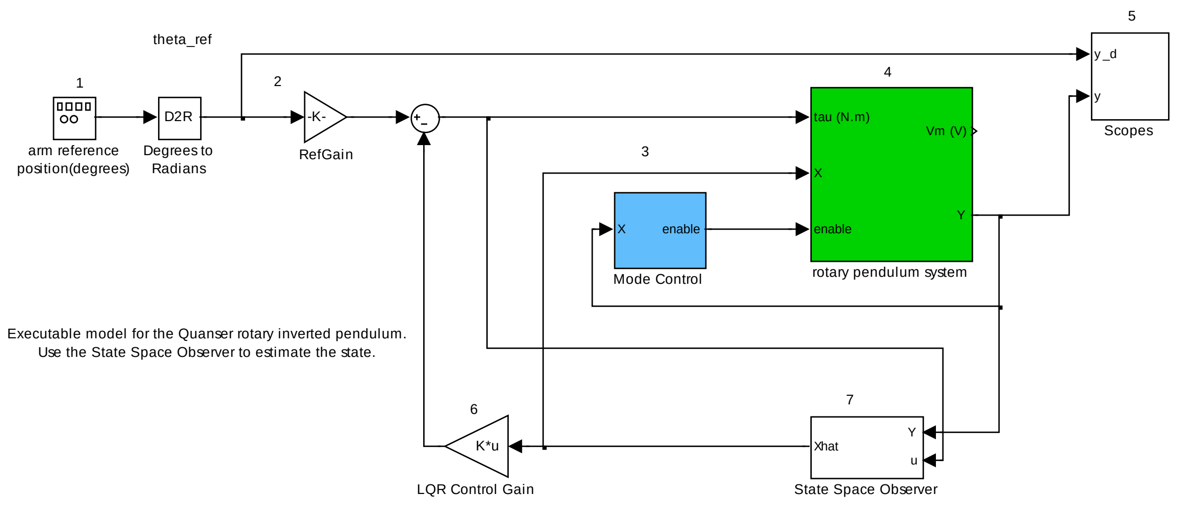


Figure 2 exe_statespace_obeserver.mdl

Block 1, subsystem 3, and subsystem 4 should be kept. Block 1 is for reference position generation. Subsystem 3 determines whether the pendulum can be controlled by the controller. Subsystem 4 contains the Q4 board accessing blocks, which read encoder values and write controls signal to the pendulum system.

7.3 Step 3

To design new controller for the QNX C++ project, users need to replace the existing Controller class. Q4 class and Reference class should be kept.

8 Conclusion

This chapter gives some experiment observations plus conclusion and possible future work.

8.1 Observations

Chapter 5 has illustrated both the simulation and execution experiments results where the pendulum is balanced. For the controller, many parameters could be tuned. Therefore, this section gives more experiment result and analyzes the observations.

8.1.1 Comparison of Results

The test bed provides simulation models for the local control system, the networked control system, and the switched system. Each system has some parameters that can be tuned. Section 8.1.1.1 shows the observations of tuning the sampling rate for local control system. Section 8.1.1.2 shows the observations of tuning the sampling rate, packet loss rate, and network type for the NCS. Section 8.1.1.3 shows the observations of tuning the switching threshold for the switched system.

8.1.1.1 Local Control Simulation

The sampling rate is an important parameter. Generally, a control system has a maximum sampling period (μ_{max}), or minimal sampling rate. Intuitively, a controller cannot work too slowly to control the inverted pendulum. To determine μ_{max} by simulation, we did several experiments where Ts was set to 5 ms, 40 ms, 20 ms, and 10 ms. The system's stability is shown in Figure 1. Since the pendulum is still stable when Ts = 10 ms (Figure 2), but is unstable when Ts = 20 ms (Figure 3). Therefore, μ_{max} must be in the range of 10 ms to 20 ms.

8.1.1.2 Networked Control Simulation

For an NCS, we can compare the sampling rate, the packet loss rate, and the performance of different network.

The first experiment is on the sampling rate of NCS. Assume that the network type is Ethernet with a data rate of 100 Mbps and no packet loss. When Ts = 5 ms, the simulation results is shown in Figure 4, in which the oscillation indicates that the LQR controller designed in Chapter 5 cannot balance the pendulum. When Ts = 4 ms, the simulation results is shown in Figure 5, which shows that the pendulum is balanced. However, compared to the pendulum angle Figure 2, which is a local control, the pendulum angle is not as smooth. And comparing the maximum pendulum angle of the two figures, we can see that the NCS is more than 2 degrees, which is larger than the pendulum angle with the local control. Therefore, this result shows that an NCS requires a higher sampling rate than a corresponding local control system.

Ts	Stability
5 ms	Stable
10 ms	Stable
20 ms	Unstable
40 ms	Unstable

Figure 1 Stability against Sampling Period

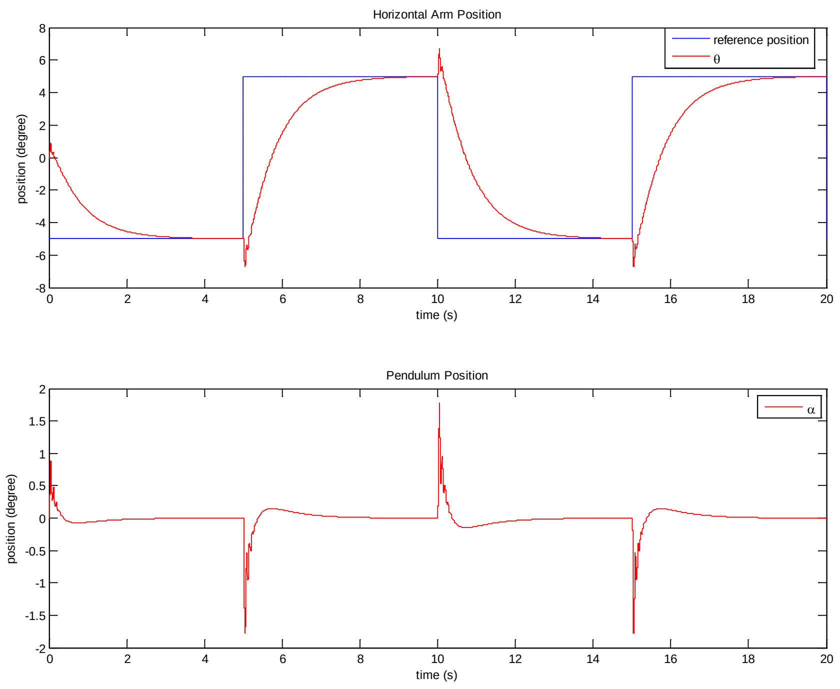


Figure 2 Local Control (Ts = 10 ms)

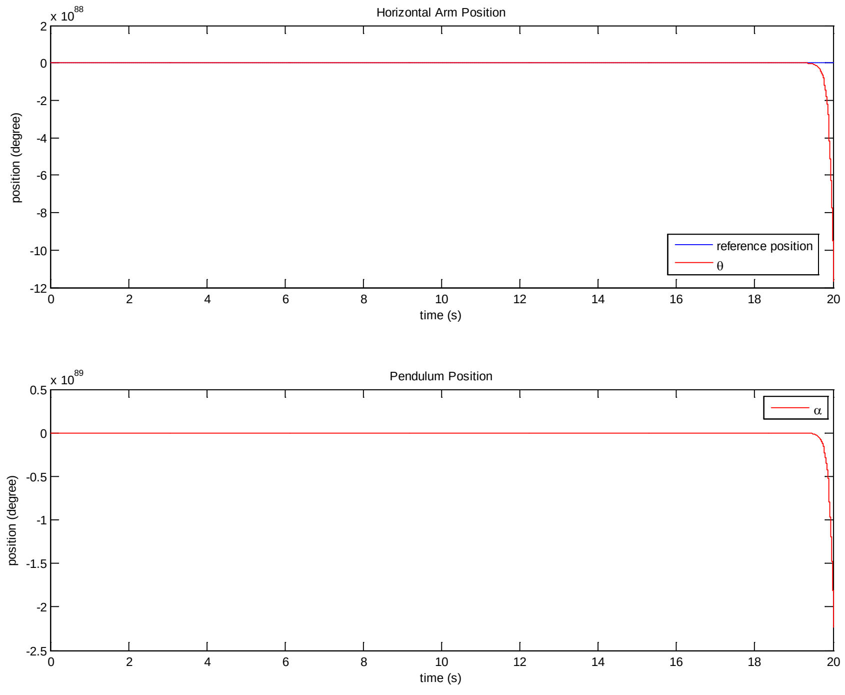


Figure 3 Local Control (Ts = 20 ms)

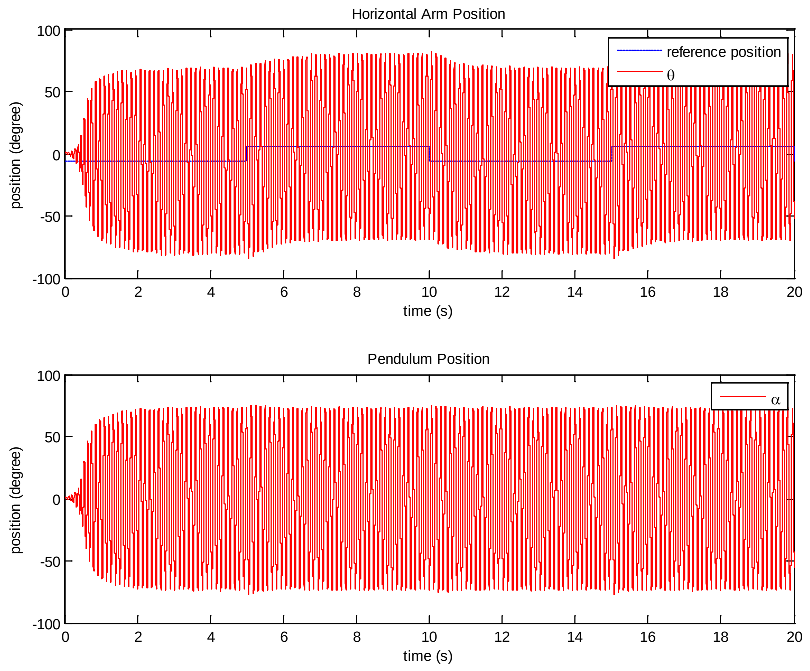


Figure 4 Network Control (Ethernet, 100Mbps, Ts = 5 ms, loss rate = 0)

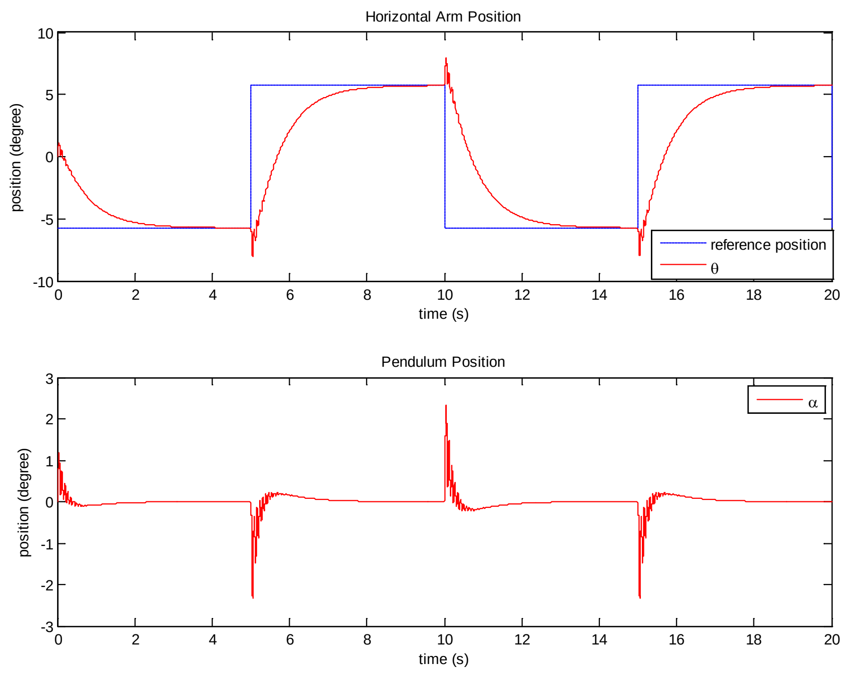


Figure 5 Network Control (Ethernet, 100Mbps, Ts = 4 ms, loss rate = 0)

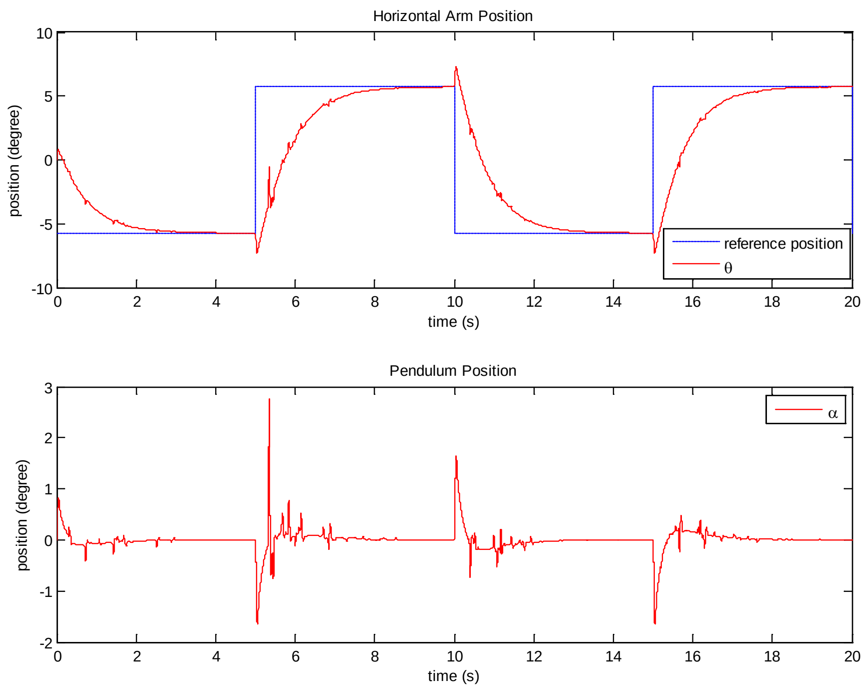


Figure 6 NCS (Ethernet, 100Mbps, Ts = 2 ms, loss rate = 2%)

TrueTime toolbox provides the ability to adjust the network parameters. The following experiments are on other network parameters. For example, we can set the packet loss rate for a TrueTime network. Still assume that the network type is Ethernet with a data rate of 100Mbps. The sensor data is transmitted to the controller every 2 ms. When the packet loss rate is 2%, the simulation result is shown in Figure 6. We can see that the pendulum angle has some random pattern compared to Figure 2 Local Control (Ts = 10 ms). This is because some packets are lost during the transmission, so that the controller cannot calculate a correct control signal. In this case, the pendulum may become unstable.

We can also set the network type. For example, assume the network type is CAN with a data rate of 1 Mbps and no packet loss. Figure 7 shows the simulation result when Ts = 4 ms. Though CAN has a much slower transmission rate (1Mpbs) than Ethernet (100Mbps), the pendulum is still balanced. This is because that CAN bus is very suitable for real time communications systems.

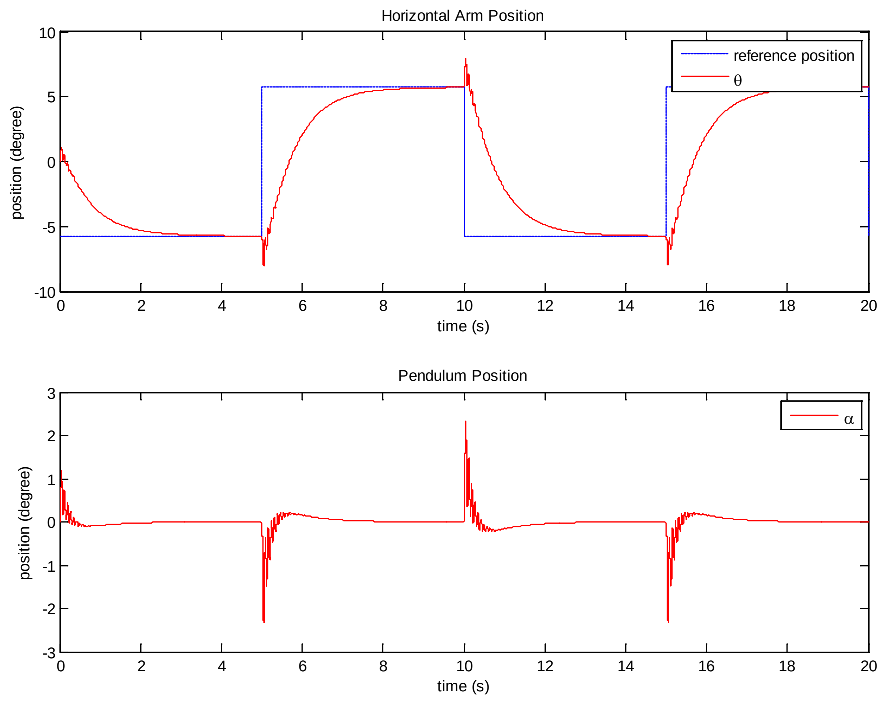


Figure 7 NCS (CAN, 1Mbps, Ts = 4 ms, loss rate = 0)

8.1.1.3 Switched System Simulation

One of our objectives has been to investigate a stability of the time scales control system, where the maximum sampling period (μ_{max}) determines the Hilger circle (with radius 1/ μ_{max}) on the complex plane, inside which the control system stays stable. Even though the system not guaranteed to be stable outside the Hilger circle, there is a conjecture that as long as the system does not stay outside the Hilger circle "too long", the system will remain stable. It should be possible to investigate this controller using the test bed, where the system would switch from inside to outside of the Hilger circle, and vice versa.

The matlab file sim_switched_system.m simulates the above idea (see in section 5.1 in Appendix C). The switching threshold is based on the norm of the difference between the pendulum angle and the reference angle, i.e., $\|\mathbf{ref}-\mathbf{x}\|$, where $\|\mathbf{x}\|$ denotes the Euclidean norm of the vector **x**. If the difference is less than the threshold, the controller will work at a longer sampling period (20 ms), otherwise it will work at a shorter sampling period (5 ms). For example, we set the switching threshold to 0.04, i.e. $T_s = \begin{cases} 5\text{ms} & \text{if } \|\mathbf{ref}-\mathbf{x}\| > 0.04 \\ 20\text{ms} & \text{if } \|\mathbf{ref}-\mathbf{x}\| \le 0.04 \end{cases}$ and the arm follows the reference signal of a square with a frequency of 0.1 Hz and an amplitude of 0.1 rad. The results are shown in Figure 8. When the reference signal changes, the controller works at a shorter sampling period, and when the reference signal is stable, the pendulum tends to work at a longer sampling period. Figure 9 shows the experimental results with a threshold 0.1, where the pendulum is still stable. However, the arm angle is less smooth compared to the arm angle in Figure 9. Therefore, this threshold controls how long the controller stays inside/outside the Hilger circle. If the threshold is too large, the pendulum may stay outside the Hilger circle too long and eventually fall.

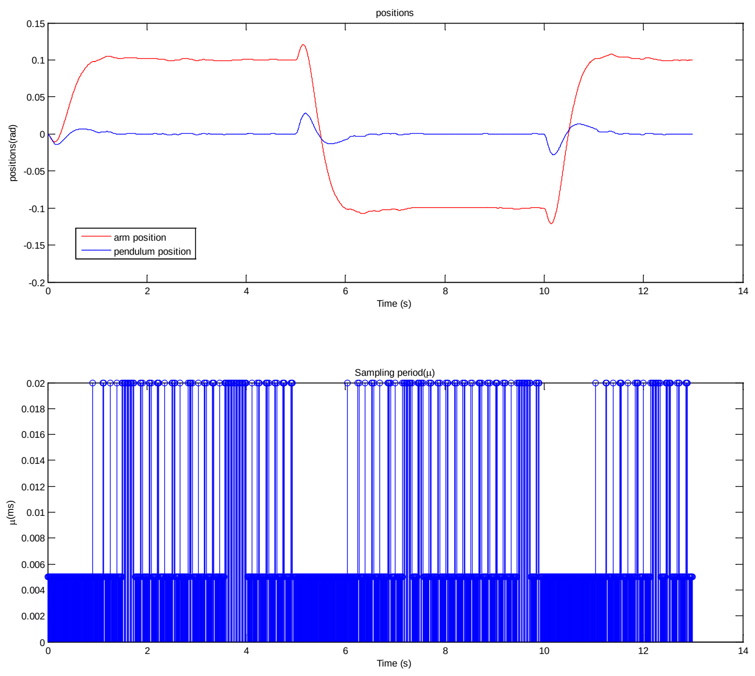


Figure 8 Switched System (threshold = 0.04)

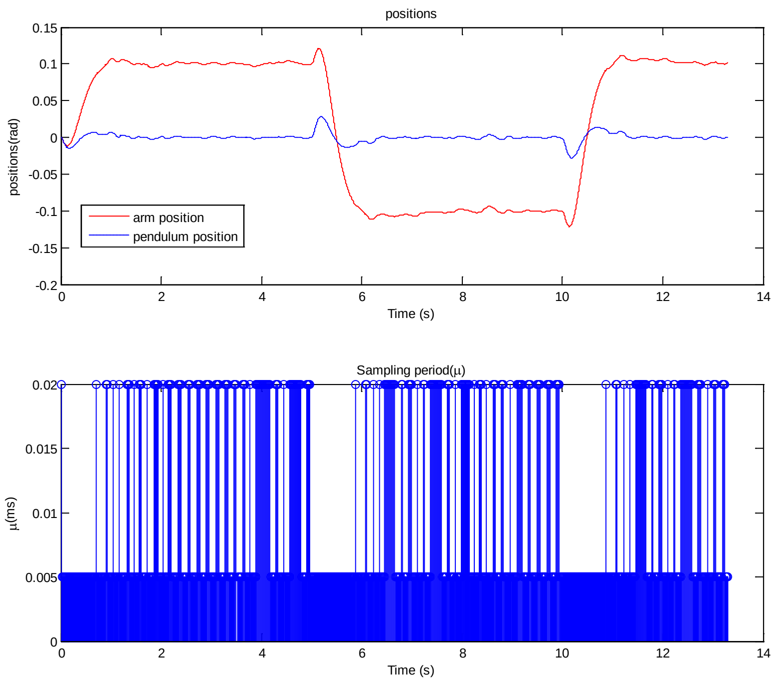


Figure 9 Switched system (threshold = 0.1)

8.1.2 Accuracy of Simulations

By comparing Fig. 11 and Fig. 19 in Chapter 4, we can see that the result is different. The simulation shows a smooth plot and no jitter, which is unlike the real experiments result. The following section discusses the potential reasons.

8.1.2.1 Mathematical Model Limitation

First, the mathematical model in equation (1) and (2) in Chapter 5 may not be accurate. The performance of the controller depends on the accuracy of the non-linear model, because the starting point of the controller design is to model the mathematical system model and get the non-linear equations of motion. The more accurate the physical model is, the more accurate the pendulum is. Besides, the controller is designed based on the state space representation of the system. It undergoes the linear approximation process of the non-linear model. With loss of information, this process also reduces the accuracy.

8.1.2.2 Motor Model

Second, the motor model also has limitations. SRV-02 incorporates a Faulhaber 2338S006 Coreless DC Motor with dynamic model shown in equation (4) in Chapter 5. The motor parameters can be found in the data sheet [1]. However, different motors could have slightly different parameters. Besides, the motor has a minimal input voltage, which means that the motor only rotates when the input is larger than this threshold. In Simulation, the horizontal arm moves whatever the input voltage is. Due to this reason, once the pendulum is near balanced position in real experiment, controller only outputs a small voltage in which case the arm will not move. After a few cycles, the pendulum will not reach the desired position. In this case, a large voltage will be generated and it pushes the arm to deviate from the balanced position.

8.1.2.3 Finite Wordlength Effect

Another reason could be the finite wordlength effect. The pendulum system uses encoders to get angle information. With a resolution of 4096, each encoder works as a 14-bit A/D converter, so that there is a certain amount of uncertainty introduced during quantization. From digital signal processing theory, we know that this will cause the finite wordlength effect, which means that some information is lost. On the contrary, Simulink simulation uses double precision variables, so that no angle information is lost. The finite wordlength effect could be analyzed using the DSP theory [2]. The simulation models can be improved to consider the finite wordlength effect.

8.1.3 Timing Inaccuracy

The C++ project uses software time to realize varying timer interrupt. Though QNX provides higher timing accuracy than Windows XP, it still has some jitter and latency. Figure 1 shows the kernel event tracing of the C++ program captured by QNX Momentics IDE. The vertical line records the time that the thread receives a message from the timer interrupt. From the figure we can see that the interval between two messages is actually 6.998 ms, not 5 ms which is preset. Besides, the interval is not fixed, ranging from 6.997 ms to 7.002ms, which indicates the jitter of the software timer.

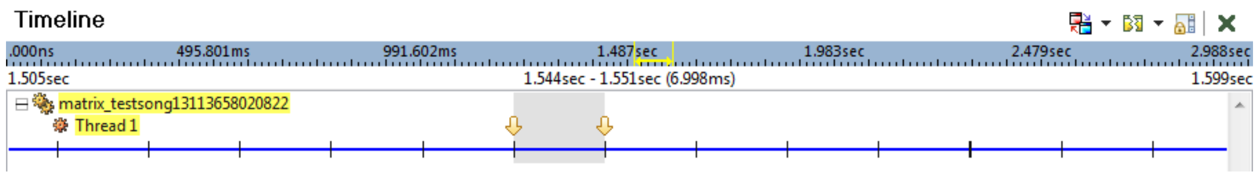


Figure 10 Kernal Event Tracing

Hardware (signal generator) provides the most accuracy timing. But currently we cannot directly control the signal generator in Simulink environment. Therefore future research could investigate how to improve the hardware to make it controllable.

8.1.4 Stability against Sampling Rate

Another experiment we did is to check the controller's stability against different sampling rate. The result shows that the controller which uses the state-space observer can balance the pendulum over a large range of sampling rate.

To generate various sampling frequencies, we use the external signal generator. The controller was designed at the sampling rate of 200 Hz. Follow Appendix C section 5.2 and configure the system to use the external signal generator. Run the controller. When the pendulum is balanced, adjust the output frequency of the signal generator. The observation is that if the frequency increased or decreased too much, the pendulum will unstable. By tuning the frequency, we find that the system is stable within 140 Hz to 240Hz. Figure 11 and Figure 12 show the arm and pendulum angles when sampling rate is 200 Hz and 240 Hz respectively. From Figure 12, we can see a larger swing of the arm, which indicates instability.

The Qanser Lab 8 (Appendix C Section 2) provides a different observer. It utilizes a differentiator and a low pass filter to estimate the velocity, which means that the observer calculates the velocity base on equation $v(n) = (s(n) - s(n-1)) / T_s$, then process $v(n)$ using a low pass filter. By repeating the above experiment, it is shown that this observer has a larger range of stability, which is 140 Hz to 300 Hz. Figure 13 and Figure 14 show the arm and pendulum angles when sampling rate is 200 Hz and 300 Hz respectively. The difference between these two models is due to the different structures. The Quanser Lab 8 provides a PD controller, while our controller is a LQR controller.

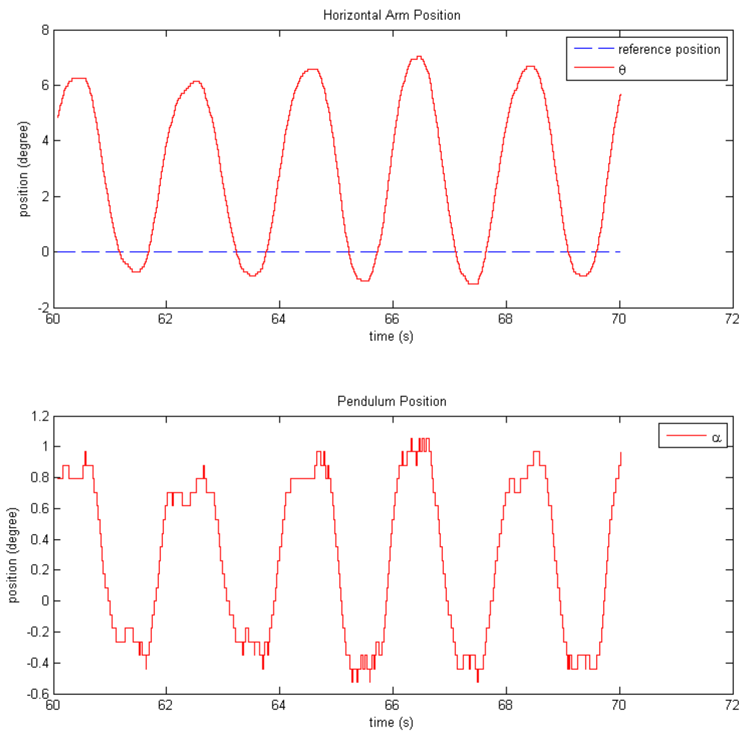


Figure 11 Running Results (Fs = 200 Hz)

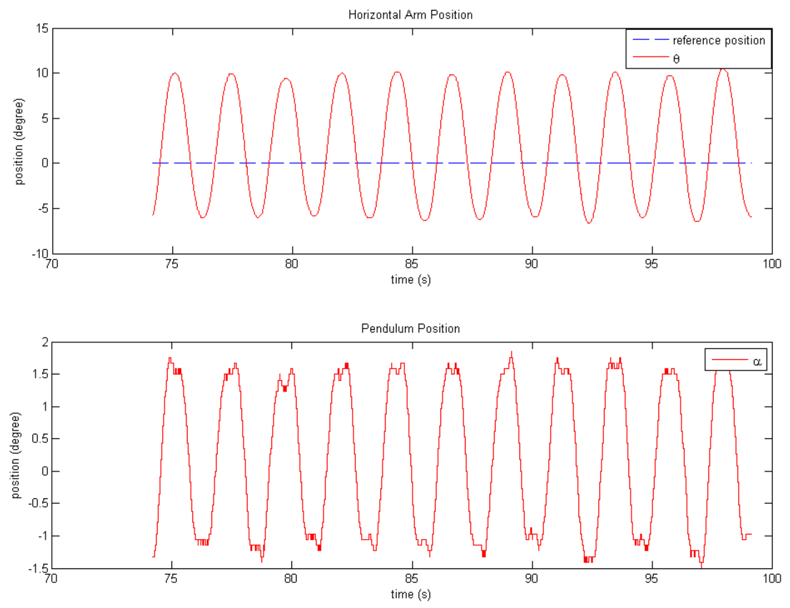


Figure 12 Running Results (Fs = 240 Hz)

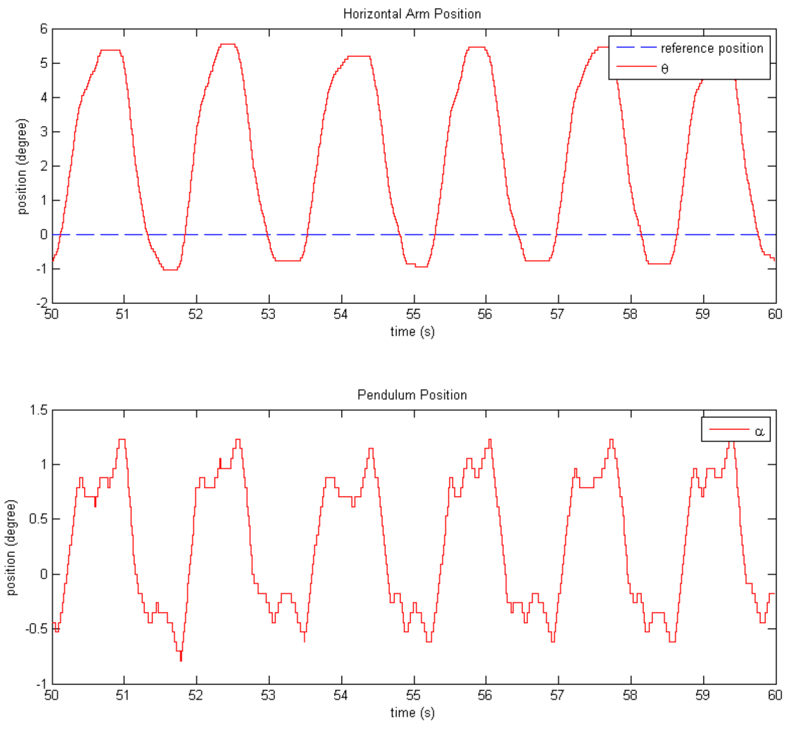


Figure 13 Quanser Lab 8 Running Result (Fs=200Hz)

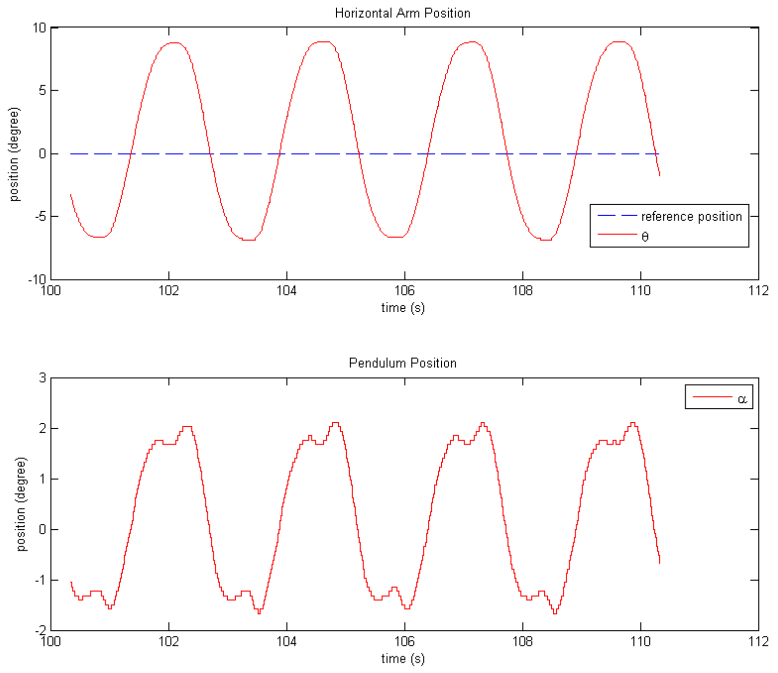


Figure 14 Quanser Lab 8 Running Result (Fs=300Hz)

8.2 Conclusions and Future Work

8.2.1 Abilities of The Test Bed

The test bed developed in this project, including the simulator and controller, could help users understand control systems. The experiments that we have done using the test bed have shown its ability to test time scales control and network control theories. More importantly, the models in the test bed provide useful blocks with interfaces that can be reused, so that uses could embed their own controllers, which is helpful for future researches.

In conclusion, the test bed developed in this project can help facilitate the future research of developing and verifying the time scales control theory as well as network control theory.

8.2.2 Limitations

Some restrictions have limited the experiments that we can do using this test bed. First, we lack the time scales control theory for a non-fullstate feedback system. For the pendulum system, where the arm and pendulum velocities are not directly sensed by sensors, it is not a full-state feedback system. The traditional pole-placement algorithm does not work quite well in this case. Second, current hardware configuration cannot send senor data directly to controller, so we have not implemented a real NCS yet.

8.2.3 Future Work

Future research could involve the following areas.

- Embed velocity sensors in the pendulum system, Add velocity sensors to get the arm and pendulum velocity, so that we can get complete state variable, and make the system a full-state feedback system.
- Develop the time scales observer. If we keep the current hardware configuration, which is not a full-state feedback system, a time scales observer is needed if we want to use the time scales control.

- Use wireless transmitters to directly transmit sensor data and control signal between the plant and the controller, which implements an NCS.
- Apply the time scales theory on NCS. One NCS co-design method is adaptive sampling period selection, which means that the controller chooses proper sampling period based on current network condition [3][4]. It has been proved that the adaptive sampling rate is feasible [4].

References

[1] Quanser Inc., "SRV02 user manual," .

[2] B. Liu, "Effect of finite word length on the accuracy of digital filters--a review," *IEEE Transactions on Circuit Theory,* vol. 18, pp. 670-677, 1971.

[3] F. Xia, L. Ma, C. Peng, Y. Sun and J. Dong, "Cross-Layer Adaptive Feedback Scheduling of Wireless Control Systems," *Sensors,* vol. 8, pp. 4265-4281, 2008.

[4] I. A. Gravagne, J. M. Davis, J. J. Dacunha and R. J. Marks, "Bandwidth reduction for controller area networks using adaptive sampling," in *Proceedings of the 2004 IEEE International Conference on Robotics and Automation,* New Orleans, LA, 2004, pp. 5250-5255.

Appendix A

Parameter Definition

The parameters used in Matlab programs are listed in the following table.

Symbol	Description	Matlab Variable	Value(SI units)
α	Pendulum angle		
$\dot{\alpha}$	Pendulum speed		
$\ddot{\alpha}$	Pendulum acceleration		
θ	Arm angle		
$\dot{\theta}$	Arm speed		
$\ddot{\theta}$	Arm acceleration		
m_p	Pendulum Mass with T-fitting	mp	0.127Kg
l_p	Distance from Pivot to Centre Of Gravity	lp	0.1556m
r	Full Length of the pendulum	r	0.2159m
J_{arm}	Moment of inertia acting seen from arm pivot	Jarm	0.0020
J_p	Moment of inertia acting seen from pendulum pivot	Jp	0.0043
B_{arm}	Equivalent Viscous Damping Coefficient	Barm	0.0024 Nms/rad
B_p	Equivalent Viscous Damping Coefficient	Bp	0.0024 Nms/rad
g	Standard gravity	g	9.81 m/s^2
η_g	Gearbox efficiency	eta_g	0.9
η_m	Motor efficiency	eta_m	0.69
K_t	Motor torque constant	Kt	7.68E-3 Nm
K_m	Back-emf constant	Km	7.68E-3 $V/(rad/s)$
K_g	Total gearbox ratio	Kg	70
R_m	Motor armature resistance	Rm	2.6Ω
I_m	Motor current		
τ_m	The torque applied at the load gear		

Appendix B

List of Hardware and Software

The hardware and software used in this project are listed in the following tables.

Hardware

Abbreviation	Name	Vender	Website
Q4	Data acquisition board	Quanser Inc.	http://quanser.com
Q4 terminal board	Q4 terminal board	Quanser Inc.	http://quanser.com
UPM-1503	Power module	Quanser Inc.	http://quanser.com
SRV02-E	Servo plant	Quanser Inc.	http://quanser.com
DI-604	Router	D-Link	http://d-link.com/default.aspx
33220A	Signal Generator	Agilent Technologies	http://www.home.agilent.com/agilent/home.jspx?lc=eng&cc=US

Software

Name	Vender	Version	Website
Matlab	The Mathworks, Inc.	R2010a	http://www.mathworks.com
QUARC	Quanser Inc.	2.1	http://quanser.com
Momentics IDE	QNX software Systems	4.7	http://www.qnx.com
QNX Neutrino RTOS	QNX software Systems	NA	http://www.qnx.com
VMware Player	WMware, Inc.	3.1.3	http://www.vmware.com
Agilent IO Libraries Suite	Agilent Technologies	16.1	http://www.agilent.com
IntuiLink Waveform Editor	Agilent Technologies	1.6	http://www.agilent.com

Appendix C

Test Bed User Manual

1 Set Up the System

1.1 Install Software

Here we assume that the Host runs Windows 7, and the Target runs QNX. Software installed on the Host includes Matlab, VMware player, the QUARC library and QNX Momentics IDE. On the target, the QUARC library is installed.

Because the QUARC library does not support Windows 7, we have to install a VMware Player on the Host so that we could run a virtual Windows XP computer, on which the QUARC library is also installed. When the QUARC library supports Windows 7, then VMware will not be necessary.

1.2 Connect Wires

The Host and Target computers communicate through an Ethernet connection. They are connected to a router, and they both obtain their respective IP addresses via DHCP. However, once the IP address of the Target is defined, this address must be manually in set in the Simulink model (see section 3.4).

The connections among the hardware components are shown in Figure 1.

- The analog output channel 0 on the Q4 terminal board (port 1) transmits DC motor control signal, and is routed to the UPM “from D/A” port (port 2).
- The UPM “to load” port (port 3) emits enough current to drive the DC motor, and is first connected to a pushdown break button, which in turn is routed to the DC motor port on SRV02 (port 4). The break button, in a blue box with a red button, is used as an emergency stop.

Normally the switch is closed. In case of emergency (e.g., pendulum rotating wildly), you can push and hold the red button to temporarily cut off electricity to stop the motor, then turn off the UPM.

- The arm encoder on SRV02 (port 6) is connected to the encoder input channel 0 (port 5) on the Q4 terminal board.
- The pendulum encoder (port 8) is connected to the encoder input channel 1 (port 7) on the Q4 terminal board.

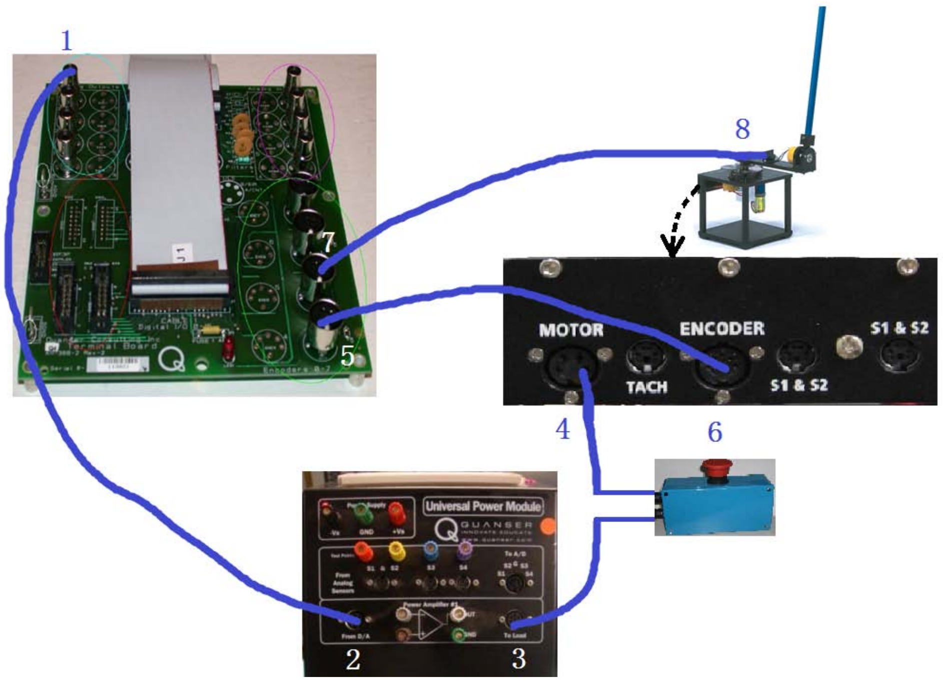


Figure 1 Wiring Diagram

1.3 Use Source Code Repository

We have used a Subversion repository to manage the files associated with this project, in conjunction with the QNX Momentics IDE. The repository contains several Matlab/Simulink simulation files, an executable Simulink model and a QNX C++ project. We can use Eclipse IDE or the QNX Momentics IDE (which is also Eclipse-based) to access the repository.

To access these files for the first time (assuming that the repository has already been created on the server), follow these steps:

(1) Create a new Eclipse workspace or use an existing workspace.

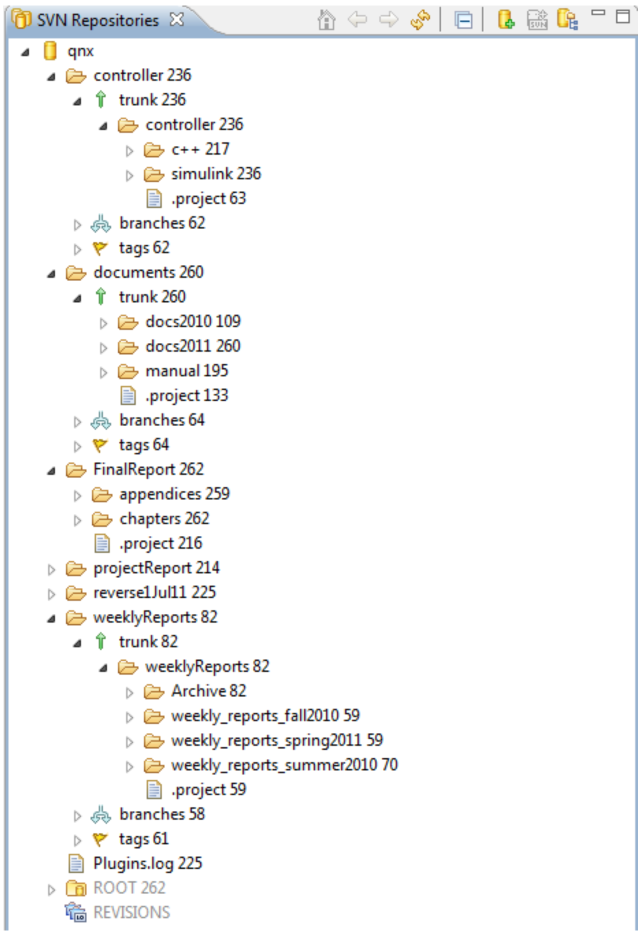
(2) Define a new repository locations within the IDE as

svn+ssh://USERNAME@wind.ecs.baylor.edu/home/grad/group/qnx/repository

and provide your user name and password to access the repository.

(3) Once connected, you should see the repository structure similar to Figure 2.

- The controller folder contains the Simulink project and the C++ project.
- The documents folder contains reports created in the project.
- The projectReport folder contains all the project reports.
- The weeklyReports folder contains Chengsen Song's weekly reports since July 2010.



```
SVN Repositories
qnx
  controller 236
    trunk 236
      controller 236
        c++ 217
        simulink 236
        .project 63
    branches 62
    tags 62
  documents 260
    trunk 260
      docs2010 109
      docs2011 260
      manual 195
      .project 133
    branches 64
    tags 64
  FinalReport 262
    appendices 259
    chapters 262
    .project 216
  projectReport 214
  reverse1Jul11 225
  weeklyReports 82
    trunk 82
      weeklyReports 82
        Archive 82
        weekly_reports_fall2010 59
        weekly_reports_spring2011 59
        weekly_reports_summer2010 70
        .project 59
    branches 58
    tags 61
  Plugins.log 225
  ROOT 262
  REVISIONS
```

Figure 2 SVN Repository

(4) Navigate and highlight the controller->trunk->controller folder to check out the controller. Right click the mouse then select check out. Other folder can be similarly checked out.

2 Run the Quanser Example (Lab 8)

Quanser provides an example (Lab 8) to demonstrate how to balance then pendulum. This section introduces how to run this Quanser example. Before running the Quanser Lab 8, log on the Host and the Target. Since the Host runs Windows 7 computer, use your user name assign by the administrator,e.g. ece01\song. For the QNX computer, log in as the super user, with the password *quarc_target*. Note that the keyboard is shared between the host and target computers. To use the keyboard on either computer, hit the “Scroll Lock” key twice to toggle between the computers.

After logging on both Host and Target, follow these steps:

(1) On the Host (running Windows 7), double click the VMware Player desktop icon to display the VMware Player window. Then Choose *open a virtual machine*. You can find the virtual Windows XP Professional located in C:\VM\XP_PRO. Then select *poweron this virtual machine*.

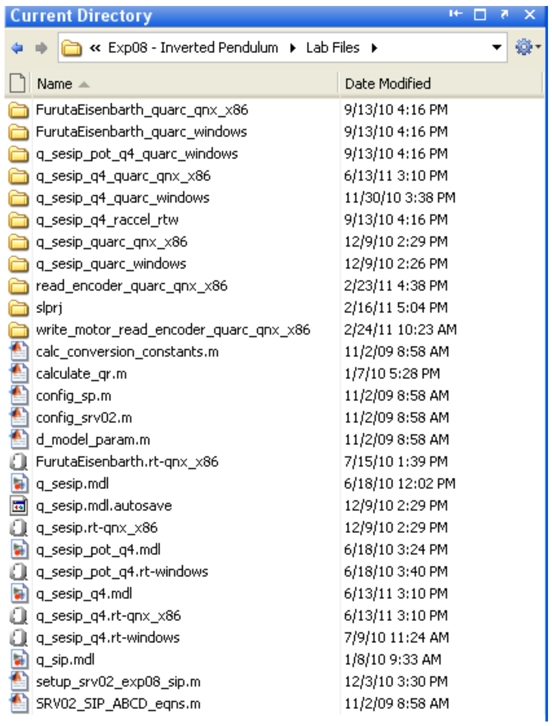
(2) On the Target, run Launch->QUARC->Console. An empty console should then appear.

(3) On the Host, switch to the Windows XP virtual machine.

(4) Start Matlab on the Windows XP virtual machine. Change the current working directory to

C:\Mprojects\MATLAB\SRV02 Lab Files\Exp08 - Inverted Pendulum\Lab Files\.

To do this, click the button in the circle shown in Figure 6. Then in the popup dialogue, find the respective folder and you should see a list similar to Figure 3.



Name	Date Modified
FurutaEisenbarth_quarc_qnx_x86	9/13/10 4:16 PM
FurutaEisenbarth_quarc_windows	9/13/10 4:16 PM
q_sesip_pot_q4_quarc_windows	9/13/10 4:16 PM
q_sesip_q4_quarc_qnx_x86	6/13/11 3:10 PM
q_sesip_q4_quarc_windows	11/30/10 3:38 PM
q_sesip_q4_raccel_rtw	9/13/10 4:16 PM
q_sesip_quarc_qnx_x86	12/9/10 2:29 PM
q_sesip_quarc_windows	12/9/10 2:26 PM
read_encoder_quarc_qnx_x86	2/23/11 4:38 PM
slprj	2/16/11 5:04 PM
write_motor_read_encoder_quarc_qnx_x86	2/24/11 10:23 AM
calc_conversion_constants.m	11/2/09 8:58 AM
calculate_qr.m	1/7/10 5:28 PM
config_sp.m	11/2/09 8:58 AM
config_srv02.m	11/2/09 8:58 AM
d_model_param.m	11/2/09 8:58 AM
FurutaEisenbarth.rt-qnx_x86	7/15/10 1:39 PM
q_sesip.mdl	6/18/10 12:02 PM
q_sesip.mdl.autosave	12/9/10 2:29 PM
q_sesip.rt-qnx_x86	12/9/10 2:29 PM
q_sesip_pot_q4.mdl	6/18/10 3:24 PM
q_sesip_pot_q4.rt-windows	6/18/10 3:40 PM
q_sesip_q4.mdl	6/13/11 3:10 PM
q_sesip_q4.rt-qnx_x86	6/13/11 3:10 PM
q_sesip_q4.rt-windows	7/9/10 11:24 AM
q_sip.mdl	1/8/10 9:33 AM
setup_srv02_exp08_sip.m	12/3/10 3:30 PM
SRV02_SIP_ABCD_eqns.m	11/2/09 8:58 AM

Figure 3 File structure for Quanser lab 8

(5) Open SETUP_SRV02_EXP08_SIP.m, which is a Matlab file used to calculate the pendulum's physical parameters and construct a PD controller for the pendulum. Run this file, and the following results should appear in the Matlab command window:

```
SRV02 model parameters:
   K   = 1.76 rad/s/V
   tau = 0.0285 s

SRV02+SIP balance control gains:
   k(1) = -0.31623 N.m/rad
   k(2) = 1.9146 N.m/rad
   k(3) = -0.14956 N.m/(rad/s)
   k(4) = 0.26268 N.m/(rad/s)
```

Here K represents steady-state gain, tau represents time constants, and k represents the feedback gain of the PD controller.

(6) Open q_sesip_q4.mdl (where sip stands for "single inverted pendulum"), which contains the Simulink model, as shown in Figure 4.

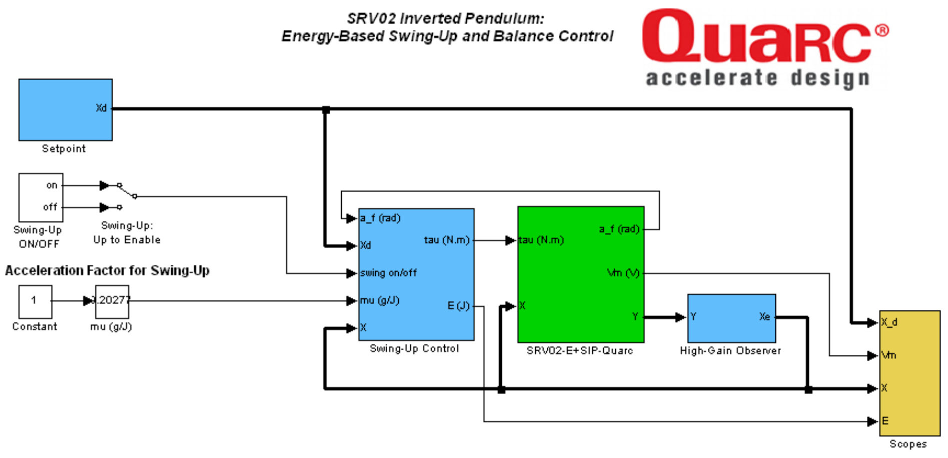


Figure 4 q_sesip_q4.mdl

(7) Build the Simulink model by selecting QuaRC->build within the Simulink window.

During the build process, the graphical representation of the Simulink model is translated into C code. The Target Language Compiler then compiles the C code and generates an executable binary specific to QNX, which is afterwards downloaded to the target.

(8) Turn on the UPM power module. The switch is on the back towards the top.

(9) Run the program on the Target. In the Simulink model window on the Host, select QuaRC->start. The controller will cause the pendulum to swing up and then balance the pendulum.

(10) Stop the program. In the Simulink model window on the Host, select QuaRC->stop.

(11) When finished, power off the power amplifier, the Windows XP virtual machine, and the QNX machine.

3 Run Our Simulink Model

We create our own Simulink model of the pendulum system, which can be used to simulate the system or actually control the inverted pendulum (when it is near the vertical position). This model can be used as the basis for designing and fine-tuning other controllers.

3.1 Introduction

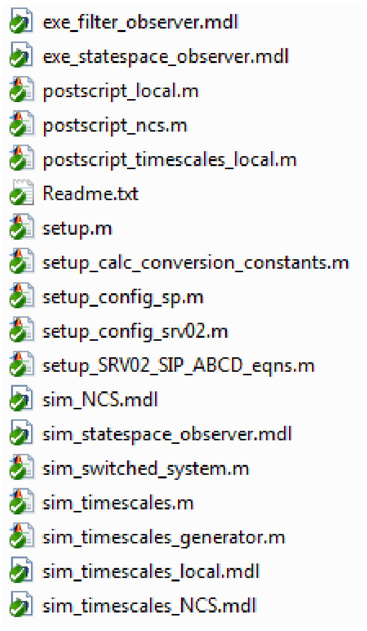
The models are located in the following address which should be stored in your personal workspace : YOUR_WORKSPACE_LOCATION\controller\simulink\quanser_pendulum_control.

The Simulink simulation model uses the LQR algorithm to control the pendulum. It is a different controller from the PD controller in Quanser Lab 8. The executable Simulink model uses the same LQR controller, which has several new blocks to directly access the hardware and handle the data.

Run the simulation model following section 3.2, which will display the pendulum and arm positions in “scopes” to indicate whether the pendulum is balanced. Note that some parameters can be tuned. You can try to tune these parameters until the results show that the pendulum can be balanced.

After the simulation results become acceptable, you can run the executable model to actually control the system. This model is compiled, and the binary code can be downloaded and run on the QNX target machine.

The relevant files are:



exe_filter_observer.mdl
exe_statespace_observer.mdl
postscript_local.m
postscript_ncs.m
postscript_timescales_local.m
Readme.txt
setup.m
setup_calc_conversion_constants.m
setup_config_sp.m
setup_config_srv02.m
setup_SRV02_SIP_ABCD_eqns.m
sim_NCS.mdl
sim_statespace_observer.mdl
sim_switched_system.m
sim_timescales.m
sim_timescales_generator.m
sim_timescales_local.mdl
sim_timescales_NCS.mdl

Figure 5 Files contained in the quanser_pendulum_control folder

The setup*.m files are Matlab files used for calculating the pendulum's physical parameters, determining control gains and other parameters that are used by the controller. The setup.m file is the main setup file, which calls the other setup_*.m files.

The .mdl files are Simulink models which can be used to simulate the system. See section 3.3 for how these files should be used.

After a simulation, the postscript*.m files can be used for drawing figures of the simulation results.

The exe*.mdl files can be compiled, downloaded, and executed on the QNX computer.

3.2 Run the Local Control Model

To run the simulation, follow these steps:

(1) Start Matlab and navigate to the current working directory. To do this, click the button in the circle shown in Figure 6. Then in the popup dialogue, navigate to :

YOUR_WORKSPACE_LOCATION\controller\simulink\quanser_pendulum_control.

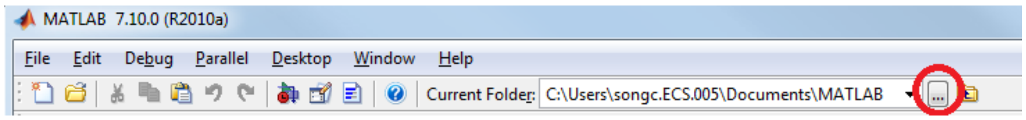

MATLAB 7.10.0 (R2010a)
File Edit Debug Parallel Desktop Window Help
Current Folder: C:\Users\songc.ECS.005\Documents\MATLAB

Figure 6 Change current working directory

(2) Choose File->Open to open the setup.m file. This file first calculates the pendulum's physical system parameters, then creates an LQR controller and the state-space observer.

You can adjust several variables to design the controller, including:

- The sampling period, which controls how fast the controller samples the system

```
%% sampling period
Ts=0.005;
```

- The Q and R matrices for the LQR algorithm

```
%% Design digital LQR controller
x = 0.2;    %weighting factor for the cart position
y = 0.4;    %weighting factor for the pendulum angle
Q = [x 0 0 0;
     0 y 0 0;
     0 0 0 0;
     0 0 0 0];
R = 5;
K_controlgain = dlqr(F,G,Q,R)
```

- The pole locations for the pole-placement algorithm

```
P = [-0.3 -0.3 -0.9 -0.9];
L = place (F',H',P)';
```

(3) Click the button in the red circle (or hit F5) to run the setup.m file. This will calculate the constants and matrices representing the pendulum system. The Matlab command window will then show the control gain matrices.

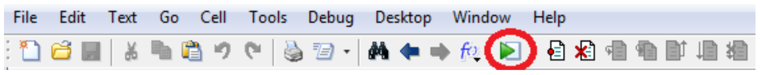


Figure 7 Run setup.m

(4) Open the sim_statespace_obeserver.mdl fiel, which represents the entire simulation model including the feedback controller, observer, and reference signal.

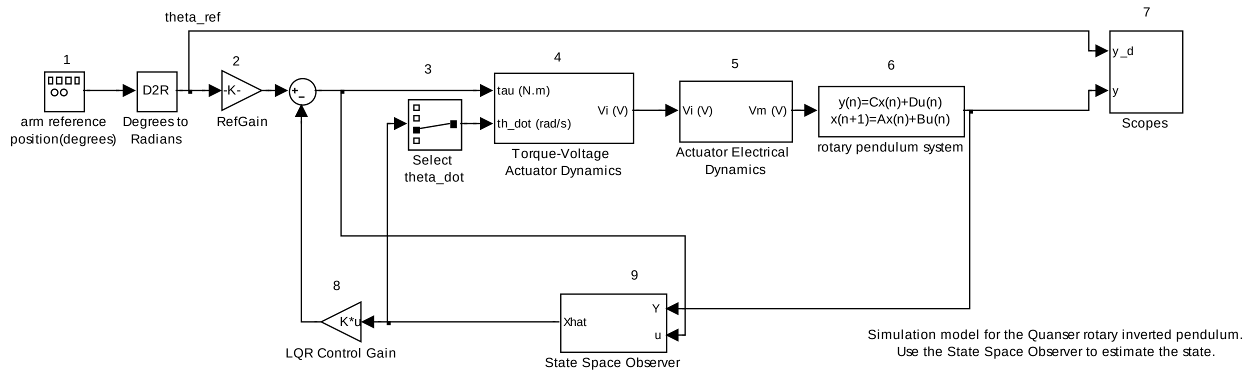


Figure 8 Simulink Simulation Model

(5) Run the model. When finished, you can see the simulation results via the Scopes. To do this, double click the Scopes subsystem. The details of this subsystem are shown in a new window, which contains two scopes. In the new window, double click both scopes to see the results.

(6) Because the output is displayed in two scopes, it is difficult to compare and analyze. We can display the results in a single figure by running the postscript_local.m file, which creates the diagram in Figure 9.

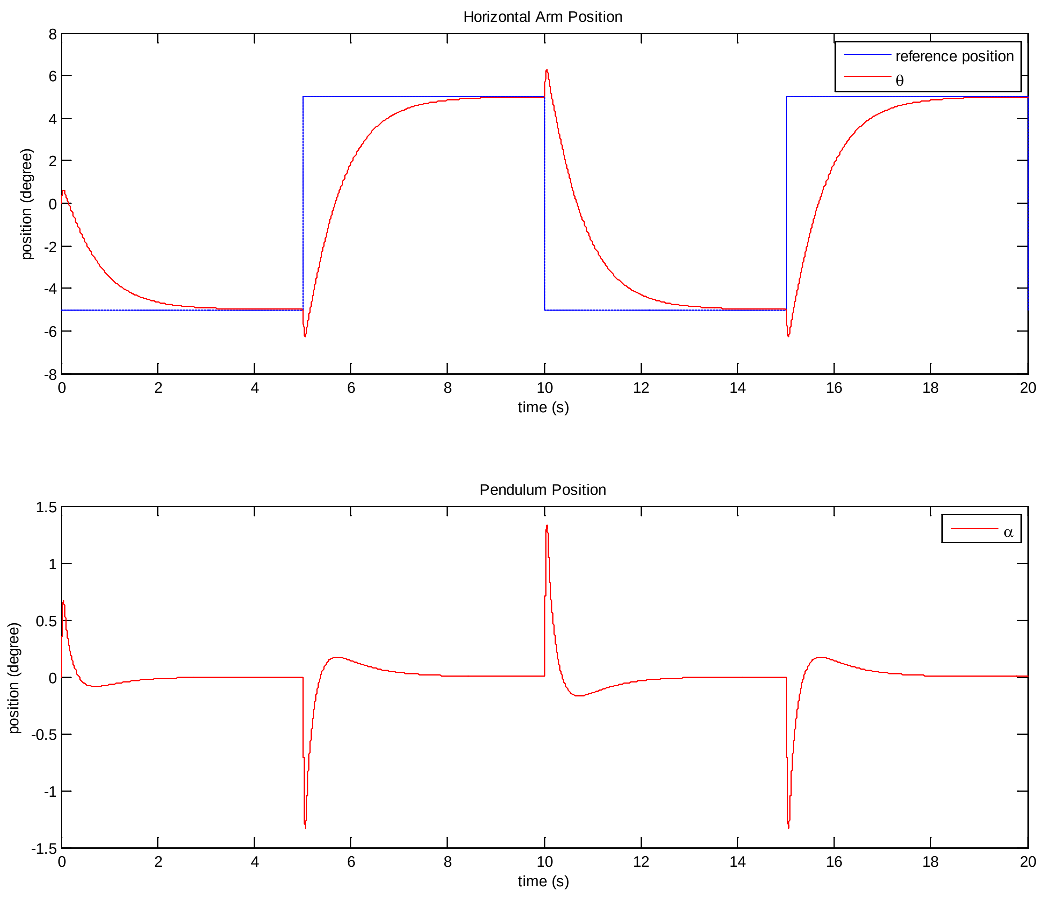


Figure 9 Local Control Simulation Result

3.3 Run the NCS Model

We created a network control system (NCS) simulation model by using the TrueTime toolbox to simulate the network. Before you run the simulation, you must first set up the TrueTime environment.

(1) Start Matlab and switch the current working directory to the following directory located in your workspace

YOUR_WORKSPACE_LOCATION \controller\simulink\truetime-2.0-beta6\examples

(2) Open the setup.m file. Edit the first line to identify the TrueTime toolbox kernel location. For example, if your workspace location is C:\Song\wks.research, then edit the first line as

```
setenv('TTKERNEL','C:\Song\wks.research\controller\simulink\truet
ime-2.0-beta6\kernel')
```

(3) Switch the current working directory to folder

YOUR_WORKSPACE_LOCATION\controller\simulink\quanser_pendulum_control

(4) Choose File->Open to open the setup.m file. For NCS, you will need a reasonably high sampling rate (or, a reasonably low sampling period). By trial and error, we found that 0.004 s is the largest sampling period that will allow the system to maintain stability. Therefore, set the sampling period to a value less than or equal to 0.004.

```
%% sampling period
Ts=0.004;
```

(5) Click the button in the red circle or hit F5 to run the setup.m file. This will calculate the constants and matrices that will be used to represent the physical characteristics of the pendulum system. The Matlab command window will show the control gain matrix.

(6) Open sim_NCS.mdl (shown in Figure 10).

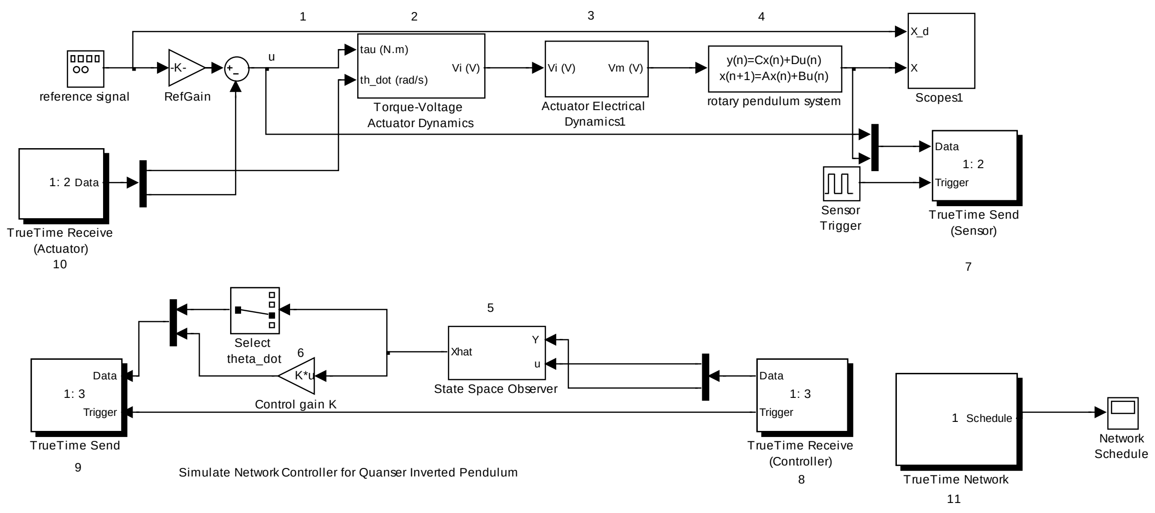


Figure 10 NCS Simulation

(7) Run the model. When finished, you can see the simulation results by double clicking the Scopes subsystem. The details of this subsystem are shown in a new window, containing two scopes. In the new window, double click each scope to see the results.

(8) As before, these plots are displayed in two scopes so that it is difficult to compare and analyze. Then open the postscript_ncs.m file and run it to display both plots in a single window (Figure 11).

If you compare Figure 9 and Figure 11, you can see that the maximum pendulum position is smaller than 1.5 degrees in the local control case, and larger than 1.5 degrees in the NCS case, due to the delay effect of the NCS.

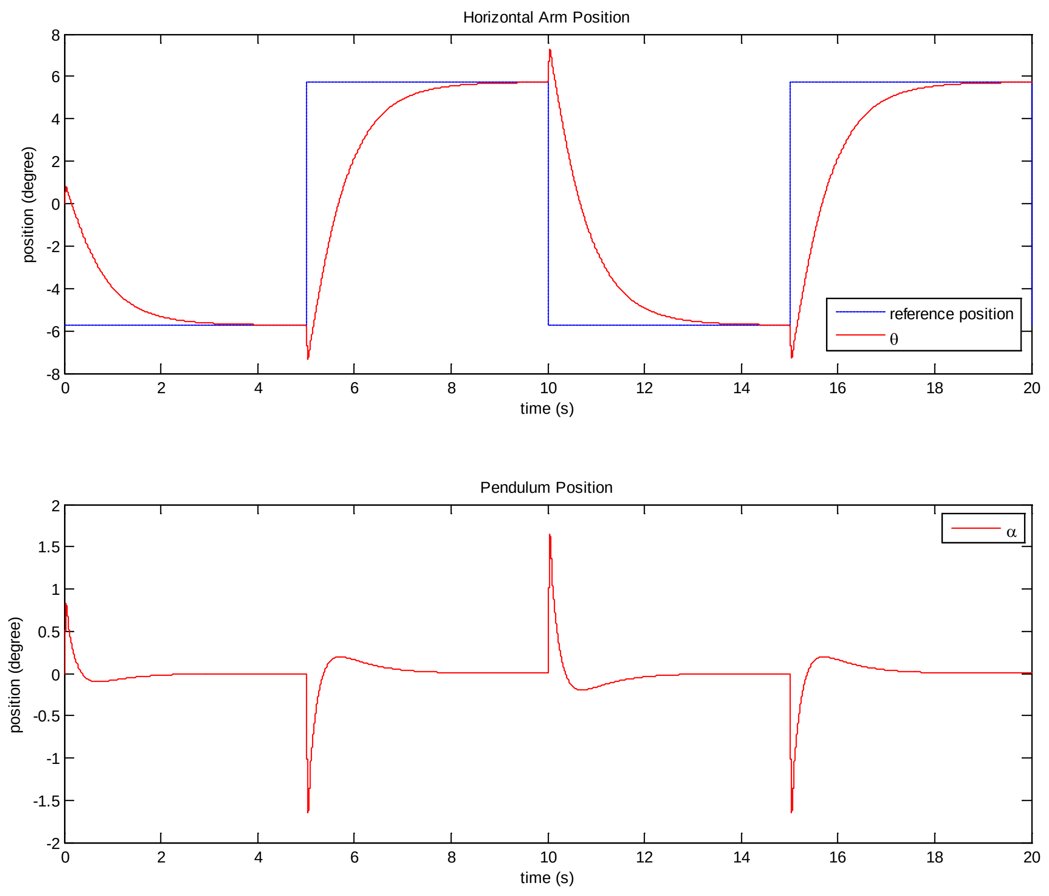
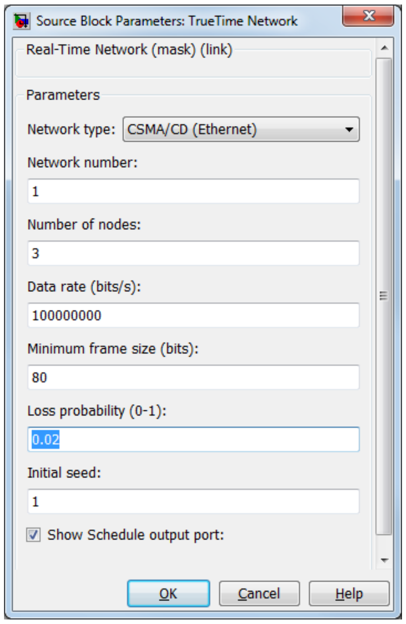


Figure 11 NCS Simulation Result

Many network parameters in the NCS model can be changed. For example, you can set the packet loss rate to see its effect. Double click the TrueTime Network block (block 11 in Figure 10). Then in the TrueTime Network Parameters dialogue, set Loss probability to 0.02. Click OK. Save the file and run the simulation. You can get a result similar to Figure 13. You can find that there are jitters in the arm and pendulum positions. However, the pendulum angle remains within 3 degrees, showing that the pendulum is still balanced.

Similarly, you can select different network types.


Source Block Parameters: TrueTime Network

Real-Time Network (mask) (link)

Parameters

Network type: CSMA/CD (Ethernet)

Network number:
1

Number of nodes:
3

Data rate (bits/s):
100000000

Minimum frame size (bits):
80

Loss probability (0-1):
0.02

Initial seed:
1

☑ Show Schedule output port:

Figure 12 TrueTime Network Parameters

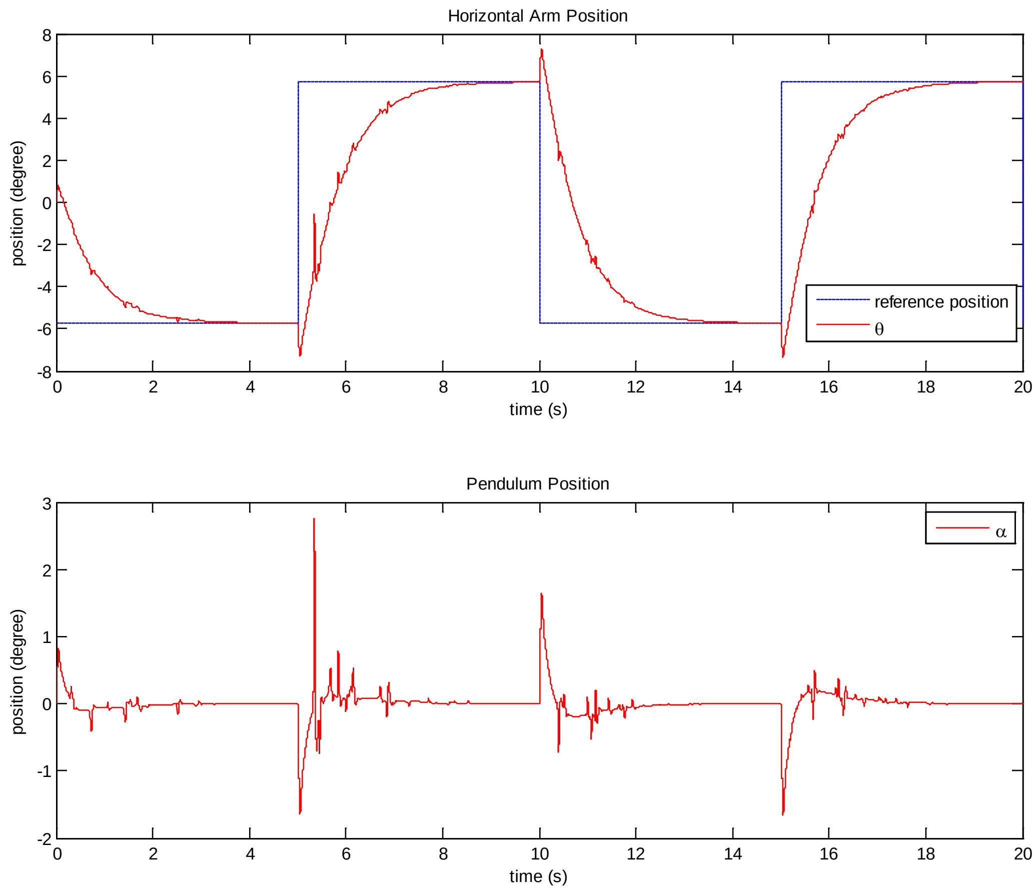


Figure 13 NCS Simulation Result (loss rate = 0.02)

3.4 Run the Local Control Executable Model

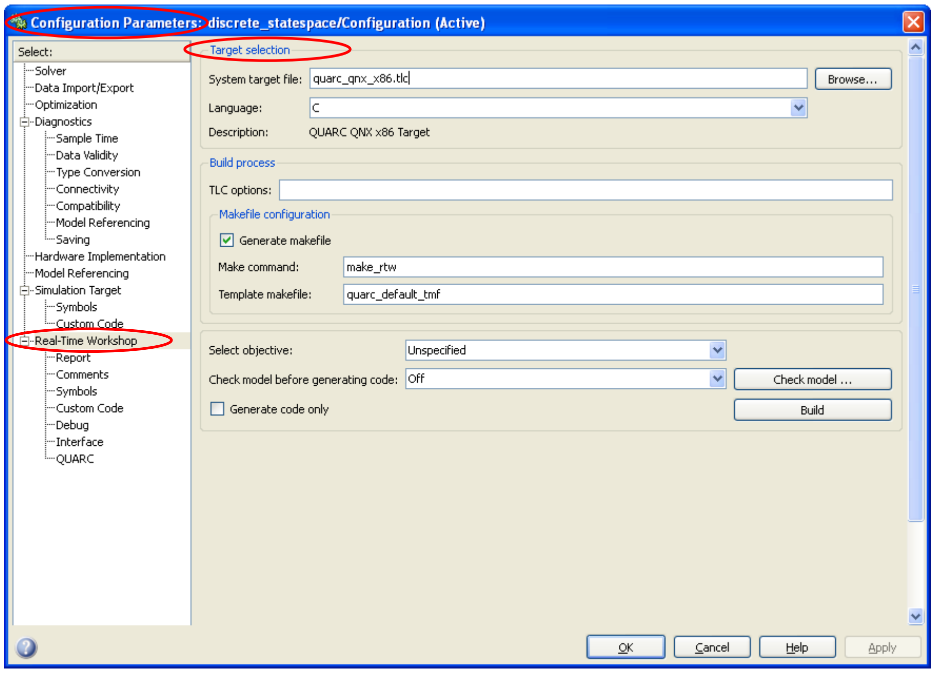
Our local control executable model implements the same controller as the local control simulation model. This model is compiled by Matlab and the QUARC library on the Host and the binary code is downloaded to the QNX machine.

Whenever you create a new executable model, you must set the target file for the model. But this step must be done only once.

(1) In the Simulink model on the Host, select QUARC->options.

(2) In the *configuration parameters* dialogue, choose *Real-Time Workshop* on the left column of the window, where you can set various simulation parameters for the model.

(3) In *target selection*, click *browse*, and choose quarc_qnx_x86.tlc in the popup dialogue (as shown in Figure 14), which identifies the kind of target to build.



Configuration Parameters: discrete_statespace/Configuration (Active)

Target selection

System target file: quarc_qnx_x86.tlc

Language: C

Description: QUARC QNX x86 Target

Build process

TLC options:

Makefile configuration

[✓] Generate makefile

Make command: make_rtw

Template makefile: quarc_default_tmf

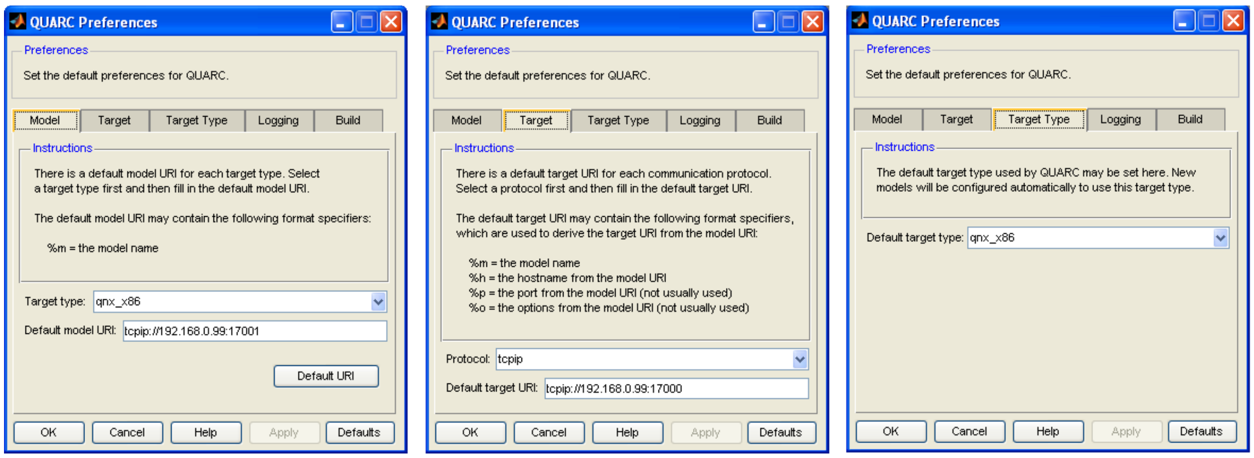
Select objective: Unspecified

Check model before generating code: Off

[ ] Generate code only

Figure 14 RTW Configuration

To set up the communicate between the Host and the Target, you need to specify the Target IP address. Switch to the Target. On the desktop, use the right mouse button and select terminal, which will display a terminal window. In the window, type *ifconfig* and press enter. The network information will be shown. Record the IP address. For example, the IP address in current configuration is 192.168.0.98. This address is fixed if the Ethernet cable remains in the same port on the router. Then switch to Host. In the Simulink model, select QUARC->preferences. In the pop up dialogue, input the information in each tab of the dialogue shown in Figure 15.



QUARC Preferences

Preferences

Set the default preferences for QUARC.

Model | Target | Target Type | Logging | Build

Instructions

There is a default model URI for each target type. Select a target type first and then fill in the default model URI.

The default model URI may contain the following format specifiers:

%m = the model name

Target type: qnx_x86

Default model URI: tcpip://192.168.0.99:17001

Default URI

OK | Cancel | Help | Apply | Defaults

QUARC Preferences

Preferences

Set the default preferences for QUARC.

Model | Target | Target Type | Logging | Build

Instructions

There is a default target URI for each communication protocol. Select a protocol first and then fill in the default target URI.

The default target URI may contain the following format specifiers, which are used to derive the target URI from the model URI:

%m = the model name
%h = the hostname from the model URI
%p = the port from the model URI (not usually used)
%o = the options from the model URI (not usually used)

Protocol: tcpip

Default target URI: tcpip://192.168.0.99:17000

OK | Cancel | Help | Apply | Defaults

QUARC Preferences

Preferences

Set the default preferences for QUARC.

Model | Target | Target Type | Logging | Build

Instructions

The default target type used by QUARC may be set here. New models will be configured automatically to use this target type.

Default target type: qnx_x86

OK | Cancel | Help | Apply | Defaults

Figure 15 Set Target IP address

Perform steps (1)-(3) in section 3.2 and compile and run the model follow these steps:

(1) On the Target, run Launch->QUARC->Console. This will run the QUARC program that will communicate with the Host.

(2) On the Host, Select menu->QUARC->compile. The Simulink model will be compiled and automatically downloaded to the QNX machine. When finished, the QNX console will prompt "---- model 'model_name' downloaded ----".

(3) Turn on the UPM power amplifier.

(4) On the Host, choose QUARC->start to run the program.

(5) Manually swing the pendulum up to the vertical position. The controller will then balance the pendulum.

(6) On the Host, select QuaRC->stop to stop the program.

(7) Turn off the power amplifier, the Host, and the Target.

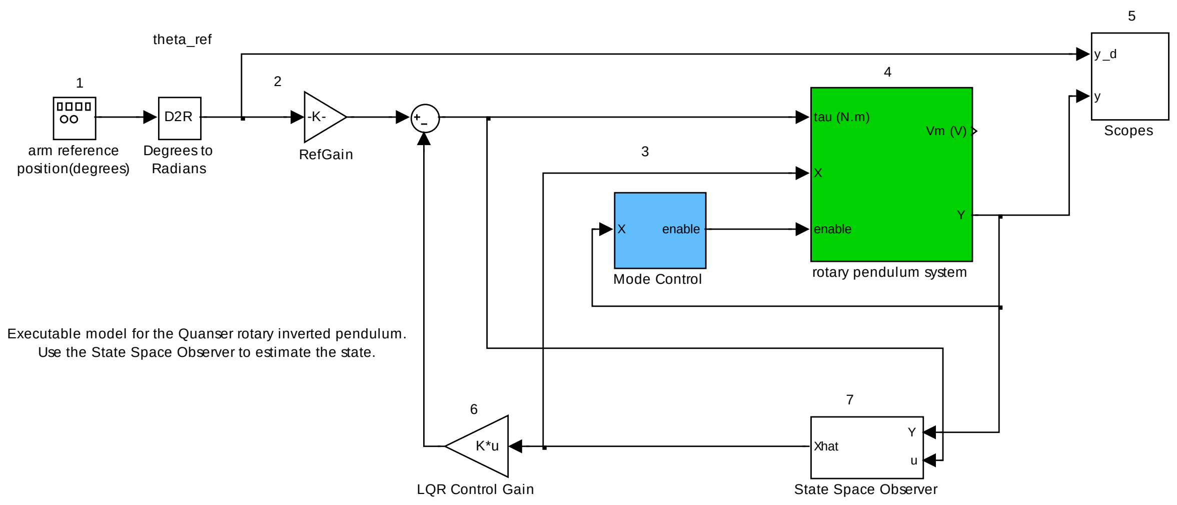


Figure 16 Simulink Executable Model

Experiments have shown that the pendulum is balanced using this balance controller (as shown in Figure 17).

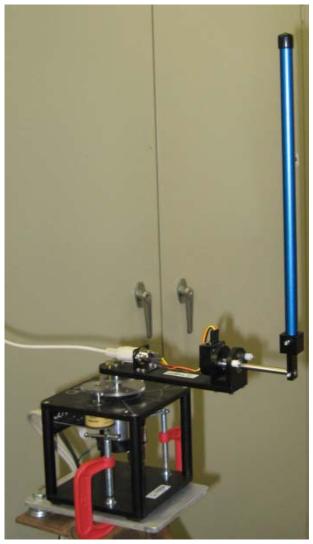


Figure 17 Experiment Result

4 QNX C++ Project

We also defined a C++ version of the controller. We used our Simulink model to define the basic architecture, and translated each Simulink subsystem into a C++ equivalent.

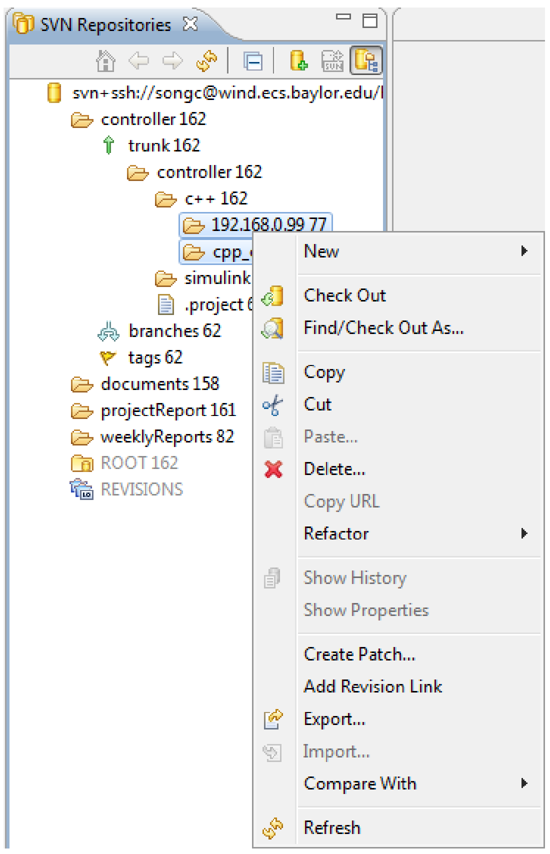
4.1 Execute the Controller

The C++ code was developed on QNX Momentics IDE on the Host running Windows 7. To check out the source code and to run the program, follow these steps:

(1) Launch QNX Momentics IDE 4.7 installed on the Host. When you run it for the first time, you must choose a workspace location. After that you can use the "current" location.

(2) Access the SVN repository. The IDE is Eclipse-based, you must define the SVN repository location, the same as the steps (1)-(3) in Section 2.3.

(3) Switch to the SVN repositories perspective after you have successfully defined the location. Browse the repository. Select the two folders in C++ folder. Right click the mouse and click check out (shown in Figure 18).



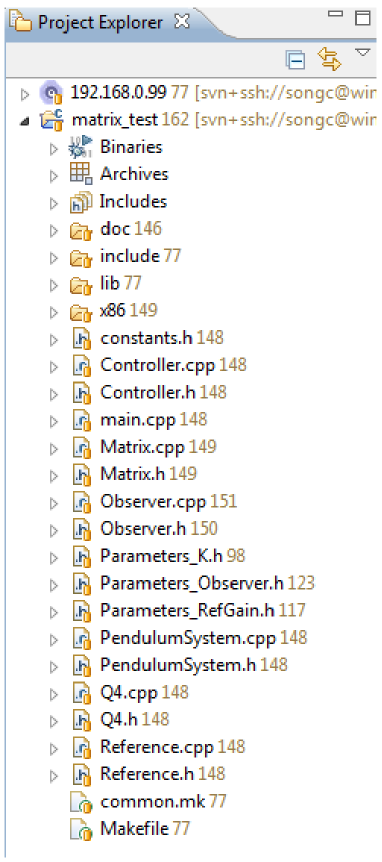


Figure 18 check out C++ project

Figure 19 Project structure

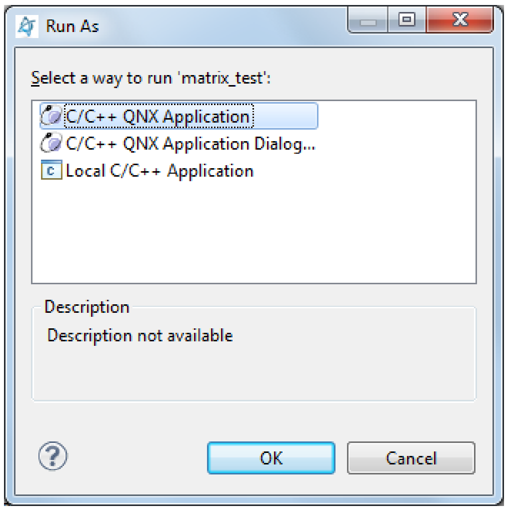
(4) After checking out the code, switch to the C++ perspective. You should see the file structure shown in Figure 19. Select the matrix_test project, and select Project->Build Project from the IDE menu to build the project.

(5) Switch to the Target. On the desktop, use the right mouse button and select terminal, which will display a terminal window. In the window, type qconn and press enter. The qconn daemon is the target agent that communicates with the host.

(6) In the Host, select Run->Run. If you run it for the first time, a dialogue will ask you to select the type of application. Select C/C++ QNX Application (Figure 20) and click OK, because the target OS is QNX. In the following dialogue (Figure 21), select the first binary file and click OK.

(7) At this point the controller should be running (Figure 22). Turn on the power amplifier, then manually swing the pendulum to the upright position. Once the pendulum is close to the vertical position, the controller begins to work to balance the pendulum.

(8) To stop the controller, click the red button in Figure 22.



Select a way to run 'matrix_test':

C/C++ QNX Application
C/C++ QNX Application Dialog...
Local C/C++ Application

Description
Description not available

Figure 20 Choose a way to run the project

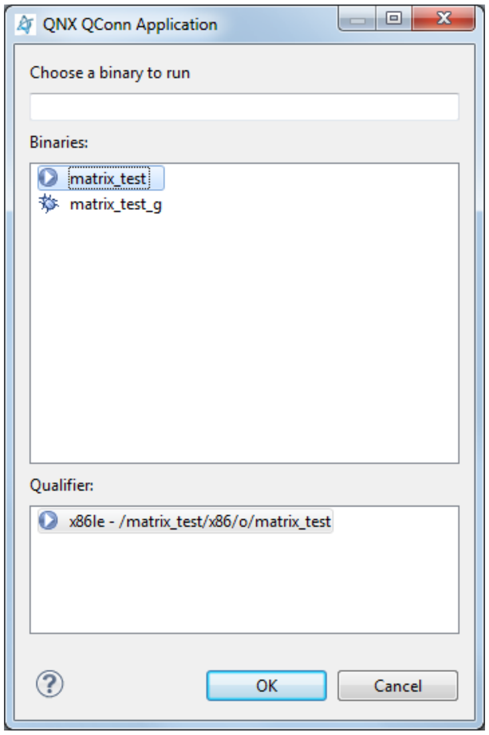
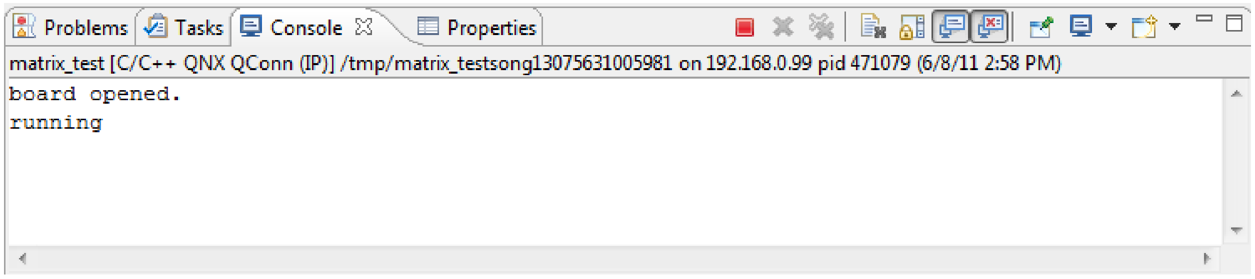


Figure 21 Choose a binary to run



```
matrix_test [C/C++ QNX QConn (IP)] /tmp/matrix_testsong13075631005981 on 192.168.0.99 pid 471079 (6/8/11 2:58 PM)
board opened.
running
```

Figure 22 Project is running

5 Varying Sampling Rate

5.1 Varying Sampling Rate Simulation

One of our objectives is to investigate the stability of the system when it switches from a stable state to an unstable state, and vice versa. We know that the Hilger circle associated with $\mu_{\max}$ determines the area that the system stays stable. However, there is a conjecture that as long as the system does not stay outside the Hilger circle “too long”, the system is still stable.

To demonstrate this, I did a simulation to determine what happened when the system switched forth and back inside and outside of the Hilger circle.

First, I use the Simulink model to find the sampling rates (1) that will balance and (2) that will not balance the pendulum. Let Ts represents the sampling period. By doing several experiments, I find that when Ts<=10ms, the pendulum is stable, and when Ts=20ms the pendulum is unstable.

Second, I investigate the case where the control system switches between the stable and unstable case. For the simulation, I create a Matlab program to model the switching mechanism, where the switch criterion is based on the Euclidian norm of difference between the reference vector and the state variable.

$$T_s = \begin{cases} 5\text{ms} & \text{if } \|\mathbf{ref}-\mathbf{x}\| > 0.04 \\ 20\text{ms} & \text{if } \|\mathbf{ref}-\mathbf{x}\| \le 0.04 \end{cases}$$

To run this simulation file, perform steps (1)-(3) in section 3.2.

Then open file sim_switched_system.m, and run it. When simulation finishes, a picture similar to Figure 23 will show the results of the arm and pendulum angle under the varying sampling rate case.

Figure 23 shows the position and Ts when the horizontal arm is tracking a square-wave reference input with amplitude 0.1 rad. Here, the above figure shows the arm position and pendulum position. We can see that the arm can track the reference position, and the pendulum angle is within 0.04 rad (2.3 degrees), indicating that the pendulum is balanced. The figure below shows the varying sampling period. Ts switches between 5ms and 20ms. When the reference input remains still, the system can be sampled at 20ms, while when the reference input changes, system must be sampled at 5ms.

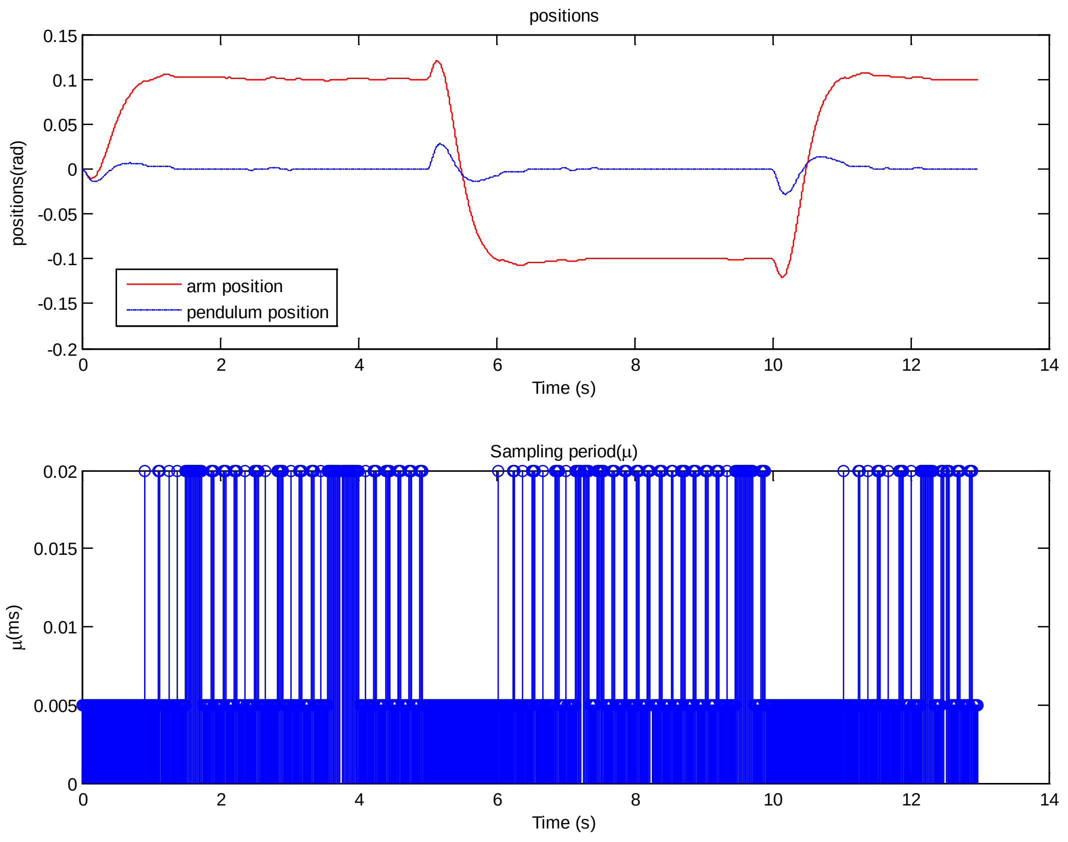


Figure 23 Switched system

In the C++ project, to test the switched system, open main.cpp, and find the following line

```
int Ts = mysys.update(true);
```

Set the parameter to false, and rebuild the project. Here update() is a member function of class PendulumSystem. The parameter controls whether the controller is working under a fixed sampling rate.

The prototype of the update() function is

```
/**
 * @brief Read encoders, calculate control signal, and write voltage out.
 * @param bool is FixedRate: indicates whether the controller is updated
 *        at a fixed rate.
 *        false - if updated at varying step length.
 *        true  - if updated at fixed step length.
 * @return the sampling period for the next step
 */
int update(bool isFixedRate);
```

5.2 Use the External Signal Generator

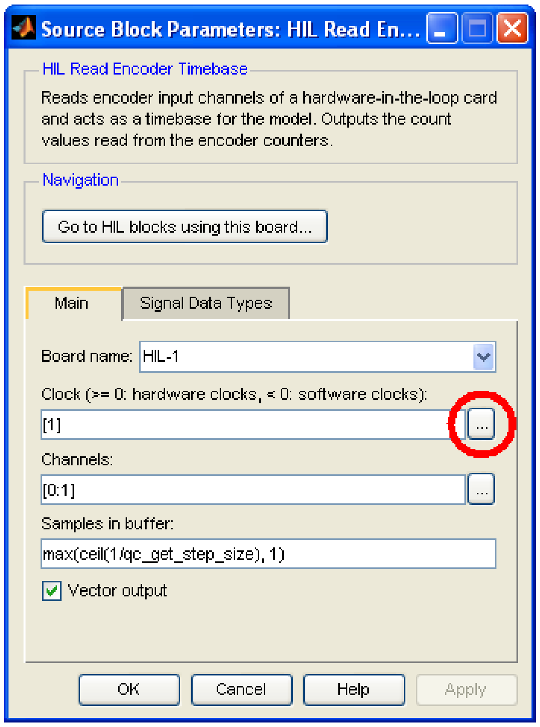
The pendulum system can be controlled by the external signal generator. To use this function, follow these steps:

(1) Use cable to connect the output of the signal generator to the input of the signal buffer, and connect the output of the signal buffer to the control port on the Q4 terminal board.

(2) Power on the signal generator, and set the output mode to square wave with frequency of 200Hz and amplitude 5 V. Vpp equals to 5 V enable the signal buffer to output valid voltage signal to the Q4 terminal board. For the detailed steps to set up the signal generator, please refer to 33220A user manual.

(3) Change the clock source. Before you compile the model, double click the rotary pendulum system subsystem. In the new window, double click the HIL Read Encoder Timebase(HIL-1) block, which pops up the Source Block Parameters dialogue (Figure 24).

(4) Change the clock channel. Click the button circled in red, which pops up the channel selection dialogue. The original channel is set to General Purpose counter (Figure 25). Select the External Interrupt (EXT_INT) channel on the left column, and click >> button to switch it to the right column, and switch the General Purpose counter to the left. The new result should be the same as in Figure 26.



Source Block Parameters: HIL Read En...

HIL Read Encoder Timebase

Reads encoder input channels of a hardware-in-the-loop card and acts as a timebase for the model. Outputs the count values read from the encoder counters.

Navigation

Go to HIL blocks using this board...

Main | Signal Data Types

Board name: HIL-1

Clock (>= 0: hardware clocks, < 0: software clocks):
[1]

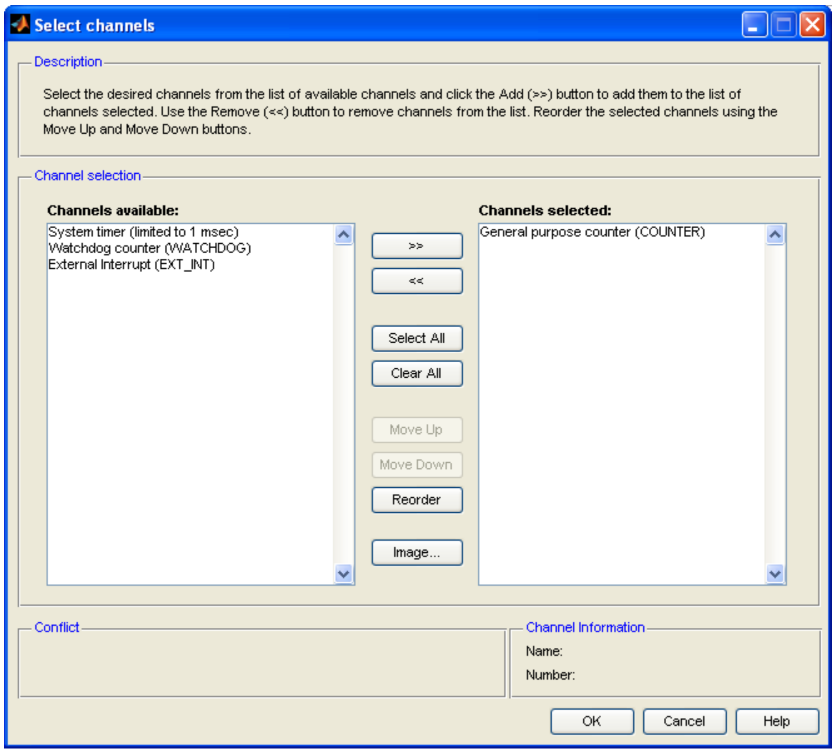
Channels:
[0:1]

Samples in buffer:
max(ceil(1/qc_get_step_size), 1)

☑ Vector output

OK | Cancel | Help | Apply

Figure 24 Source Block Parameters for the HIL Read Encoder



Select channels

Description

Select the desired channels from the list of available channels and click the Add (>>) button to add them to the list of channels selected. Use the Remove (<<) button to remove channels from the list. Reorder the selected channels using the Move Up and Move Down buttons.

Channel selection

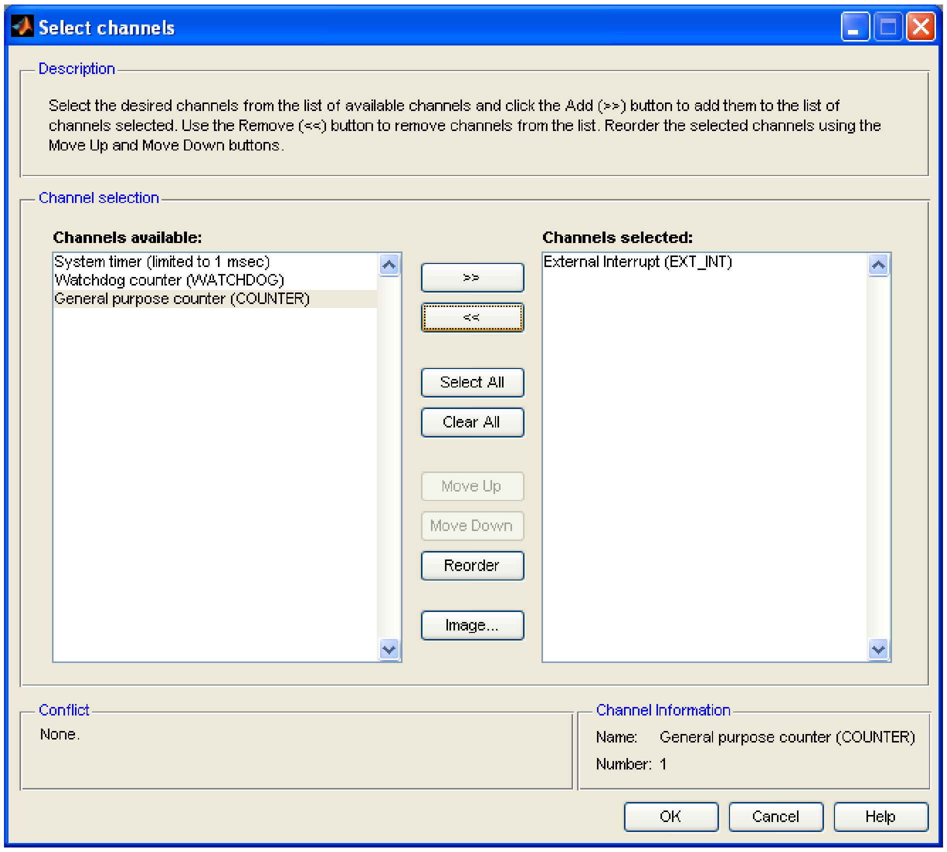
Channels available:
System timer (limited to 1 msec)
Watchdog counter (WATCHDOG)
External Interrupt (EXT_INT)

Channels selected:
General purpose counter (COUNTER)

Conflict

Channel Information
Name:
Number:

Figure 25 Select channels


Select channels

Description

Select the desired channels from the list of available channels and click the Add (>>) button to add them to the list of channels selected. Use the Remove (<<) button to remove channels from the list. Reorder the selected channels using the Move Up and Move Down buttons.

Channel selection

Channels available:
System timer (limited to 1 msec)
Watchdog counter (WATCHDOG)
General purpose counter (COUNTER)

Channels selected:
External Interrupt (EXT_INT)

Conflict
None.

Channel Information
Name: General purpose counter (COUNTER)
Number: 1

Figure 26 Select external interrupt as the new clock channel

(5) Repeat the steps as described in section 3.4.

(6) When the controller is running, the Simulink model is updated at the frequency of the external signal generator. You can slightly adjust the output frequency of the signal generator to see the performance of the controller.

Appendix D

How to generate arbitrary time scales

This appendix introduces how to generate arbitrary time scales using the Agilent 33220A signal generator [1]. Before start, you need to install the Agilent IO Libraries Suite 16.1 [2] and IntuiLink Waveform Editor [3] on the Host. Agilent 33220A is required to be connected to the router where the Host and Target are connected. In current configuration, Agilent 33220A will obtain IP address by DHCP.

1. Power on Agilent 33220A.
2. Power on the Windows 7 Host machine. Run Start->All Programs->Agilent IO Libraries Suite-> Agilent Connection Expert. A window similar to Figure 1 is shown.

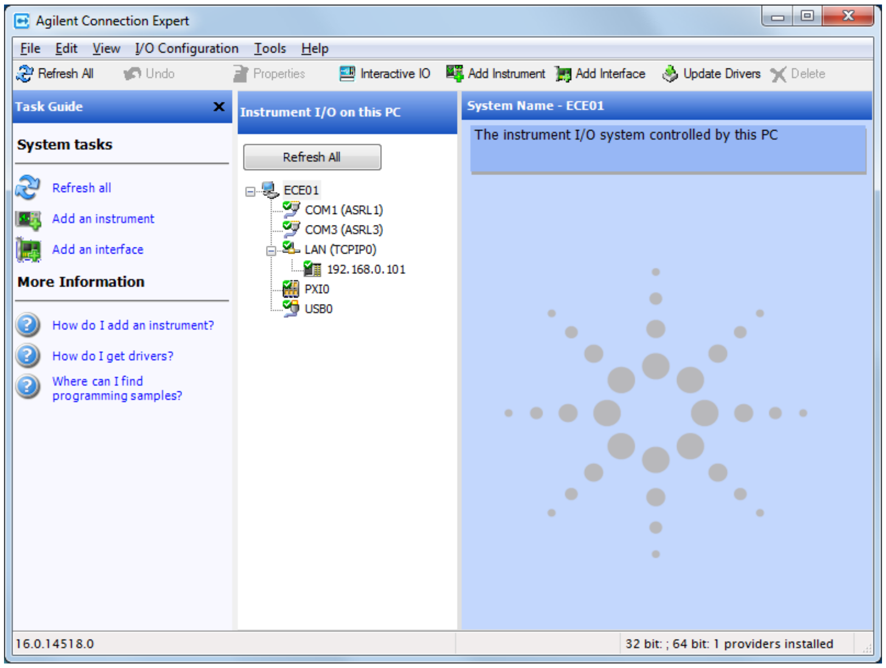
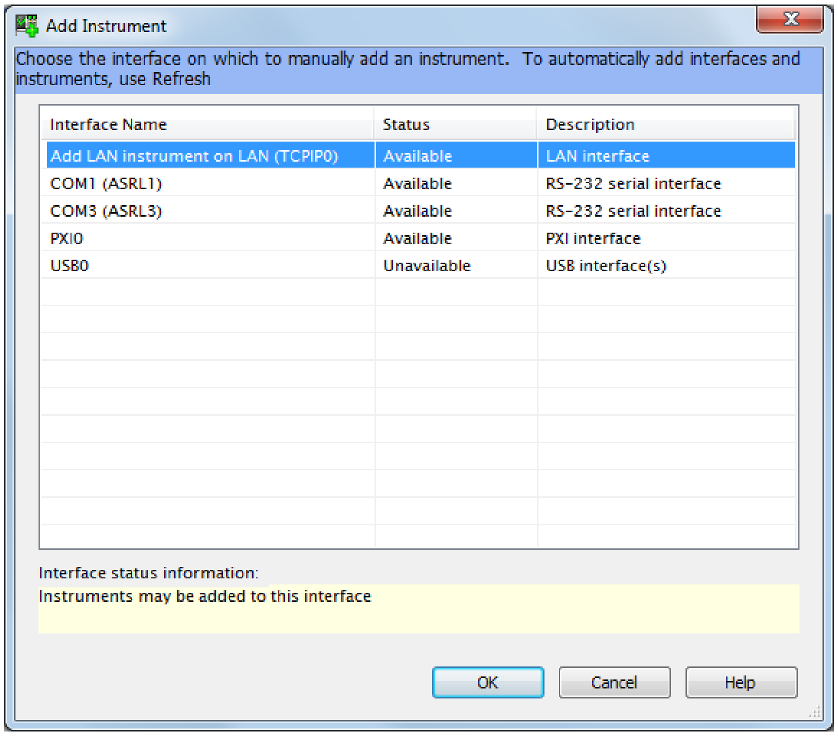


Figure 1 Agilent Connection Expert

3. Click Add Instrument, and pop up the dialogue shown in Figure 2. Click OK. The software will search for instruments on local Ethernet.



Add Instrument

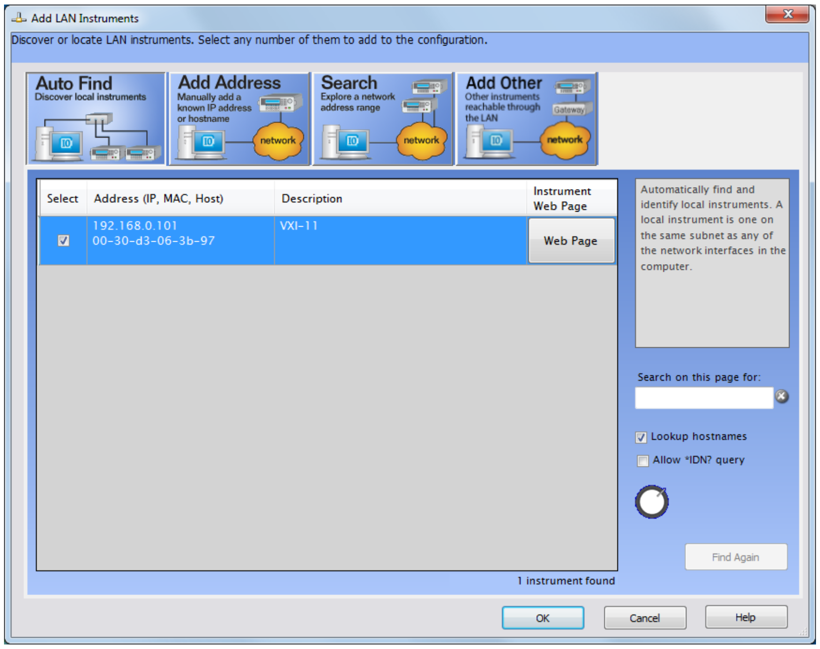
Choose the interface on which to manually add an instrument. To automatically add interfaces and instruments, use Refresh

Interface Name	Status	Description
Add LAN instrument on LAN (TCPIP0)	Available	LAN interface
COM1 (ASRL1)	Available	RS-232 serial interface
COM3 (ASRL3)	Available	RS-232 serial interface
PXI0	Available	PXI interface
USB0	Unavailable	USB interface(s)

Interface status information:
Instruments may be added to this interface

Figure 2 Add Instrument

4. Once the instrument is found, a dialogue will show (Figure 3). Click OK.



Add LAN Instruments

Discover or locate LAN instruments. Select any number of them to add to the configuration.

Auto Find
Discover local instruments
Add Address
Manually add a known IP address or hostname
Search
Explore a network address range
Add Other
Other instruments reachable through the LAN
Gateway
network

Select	Address (IP, MAC, Host)	Description	Instrument Web Page
☑	192.168.0.101 00-30-d3-06-3b-97	VXI-11	Web Page

Automatically find and identify local instruments. A local instrument is one on the same subnet as any of the network interfaces in the computer.

Search on this page for:

☑ Lookup hostnames
☐ Allow *IDN? query

1 instrument found

Figure 3 Instrument found

5. Run Start -> All Programs -> Agilent IntuiLink -> Waveform generator -> Waveform editor. This software is used to design waveform that will be generated by the signal generator.

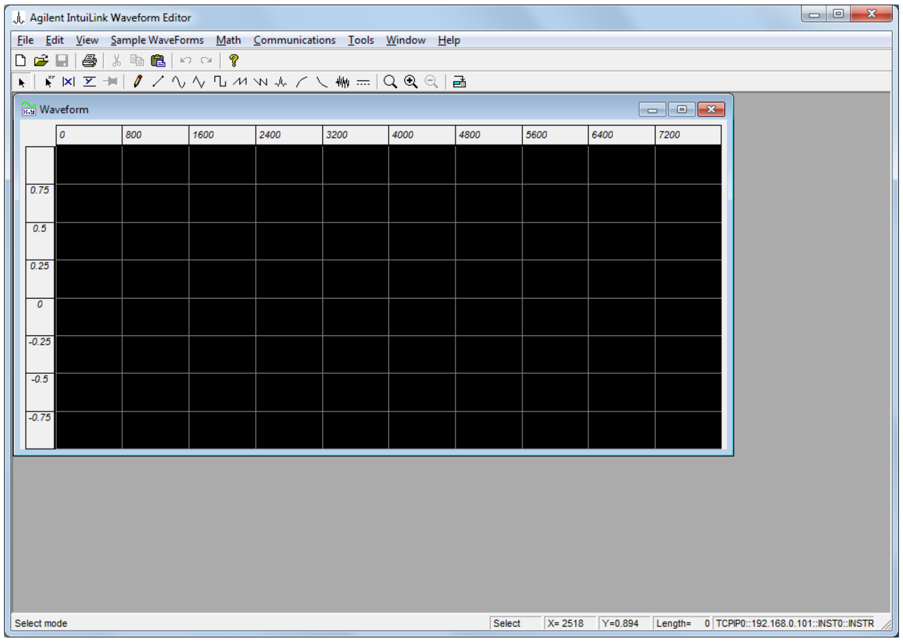
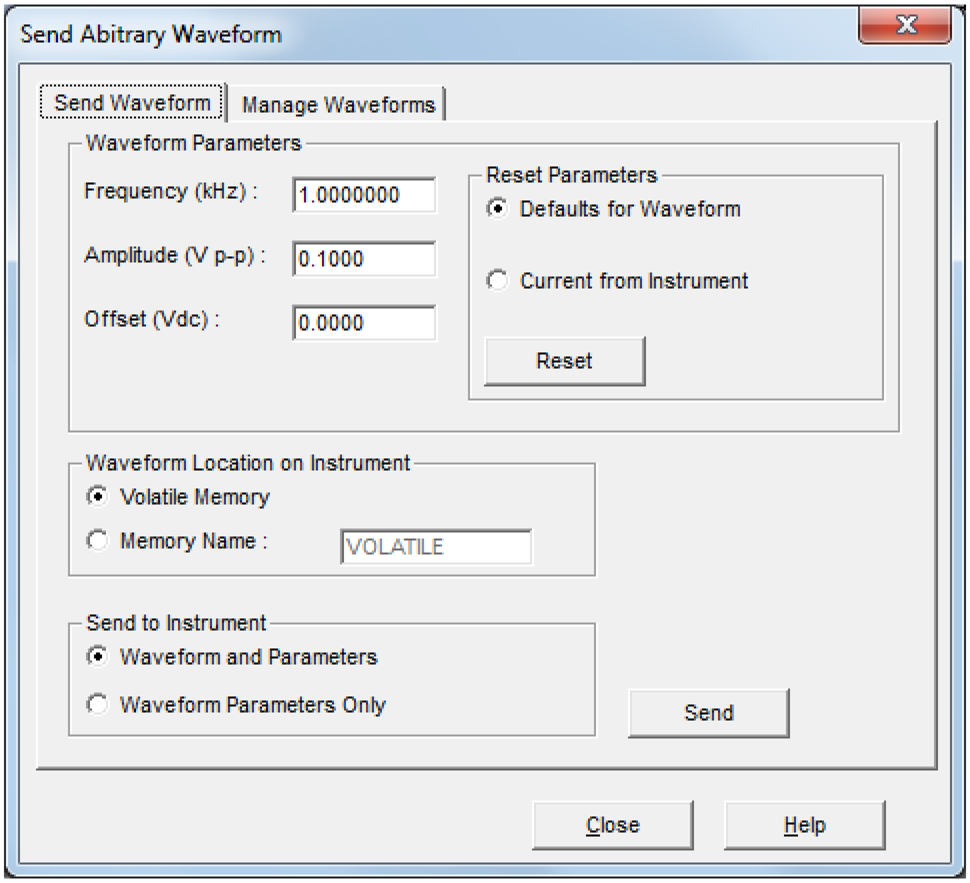

Agilent IntuiLink Waveform Editor
File Edit View Sample WaveForms Math Communications Tools Window Help
Waveform
0
800
1600
2400
3200
4000
4800
5600
6400
7200
0.75
0.5
0.25
0
-0.25
-0.5
-0.75
Select mode
Select
X= 2518
Y=0.894
Length= 0
TCPIP0::192.168.0.101::INST0::INSTR

Figure 4 Run Waveform editor

6. Edit wave form using the graphic tool. After finishing editing, you can select to save the waveform.
7. On the 33220A, push the “Arb” button.
8. In Waveform editor, select Communication -> Send waveform. A dialogue will show (Figure 5). You can edit the parameters.



Send Abitrary Waveform

Frequency (kHz) : 1.0000000

Amplitude (V p-p) : 0.1000

Offset (Vdc) : 0.0000

Reset Parameters: [✓ Defaults for Waveform] Current from Instrument

Waveform Location on Instrument: [✓ Volatile Memory] Memory Name : VOLATILE

Send to Instrument: [✓ Waveform and Parameters] Waveform Parameters Only

Figure 5 Send Arbitrary Waveform

9. Edit parameters and click Send button. A dialogue will prompt the estimated transferring time. Click OK.
10. The edited waveform will be sent to the signal generator through Ethernet. After the transmission is done, the signal generator will output the desired waveform.

References

[1] Agilent Technologies. *Agilent 33220A User's Guide* [online]. Available: http://cp.literature.agilent.com/litweb/pdf/33220-90002.pdf.

[2] Agilent Technologies. IO libraries suite 16.1. [online]. Available: http://www.home.agilent.com/agilent/product.jspx?cc=US&lc=eng&nid=-34466.977662&&cc=US&lc=eng.

[3] Agilent Technologies. IntuiLink waveform editor. [online]. Available: http://www.home.agilent.com/agilent/editorial.jspx?ckey=1000000918:epsg:sud&id=1000000918:epsg:sud&nid=-536902257.536881980.02&lc=eng&cc=IN.